COLLEGE ALGEBRA:

AN INDIVIDUALIZED APPROACH

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J. B.

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CHAPTER I

INTRODUCTION

College algebra is a course which presents unique problems to the mathematics department. The students who enroll in it are generally not mathematics majors, so there is a wide range of mathematical ability. It is quite common to find students in this class who have had only one year of algebra in high school, and who later discovered they needed more mathematics as background material for their major field. As a general rule, these students have forgotten nearly everything they ever knew about algebra. Some of them catch on right away, but it remains foreign to many others. With such a variety of students, it is difficult to find a single method of teaching which will meet their individual needs. An individualized approach to this course, using behavioral objectives, seemed a very good way to meet this problem. This thesis includes a discussion of behavioral objectives and their application to this college algebra course.

A behavioral objective has specific properties which distinguish it from other objectives. A behavioral objective describes how a student should behave after completing an educational experience. It describes the behavior the student should display and the materials he will have to work with. It is stated specifically so that the behavior can be recognized.[1] The theory is that no matter what the objective is

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[1]Elliot W. Eisner, "Instructional and Expressive Educational Objectives: Their Formulation and Use in Curriculum," Instructional Objectives (AERA Monograph Series on Curriculum Evaluation, No. 3. Chicago: Rand McNally & Co., 1969), p. 1.

there is no way of determining if it has been achieved unless there is an observable, overt behavior on the part of the learner.[2] Thus the word behavioral is used. The purpose of a behavioral objective is to reliably communicate the intent of instruction through the use of descriptions of observable behavior.[3]

The term was first used by behavioral psychologists whose learning theories were developed by studying methods of operant conditioning of animals. Here the terminal behavior itself is the goal of the learning process. Principles established here were adapted to training people for specific tasks where the goal of learning was also attainment of the terminal behavior. This was successful and lead to the "closed-loop" learning sequence with provisions for recycling the individual if learning did not take place the first time.[4]

However this "closed-loop" learning sequence does not apply to every educational situation. As Eisner points out there is a dual concern of education: to help students become skilled in the use of the cultural tools already available such as reading, writing, and arithmetic; and to help students modify and expand these tools by providing opportunities for them to construe their own interpretations

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[2] A. F. Eiss and M. B. Harbeck, Behavioral Objectives in the Affective Domain (Washington, D. C.: National Science Supervisors Association, 1969), p. 5.

[3] Henry Walbesser, Constructing Behavioral Objectives (College Park, Maryland: Bureau of Educational Research and Field Services, University of Maryland, 1970), p. 11.

[4] Eiss and Harbeck, op. cit., p. 6.

to material which they encounter.[5] Thus three domains are recognized for which behavioral objectives can be written: psychomotor, cognitive, affective.[6]

The psychomotor domain involves motor skills such as walking, writing, or typing. This type of objective is often associated with vocational training.

The cognitive domain deals with knowledge and understanding. Here objectives unambiguously specify the particular behavior the student should acquire. Activities are selected which should be useful in helping the students to attain the behavior, and evaluation determines whether it has been achieved. An effective curriculum using behavioral objectives in the cognitive domain develops behavior which all students will acquire, although not always at the same point in time. In other words, the terminal behaviors of the student are isomorphic to the objectives.[7]

The affective domain is concerned with values, attitudes and inter of the students. It involves the more sophisticated goals of education such as developing the ability to think and reason, and encouraging creativity.[8] Objectives in this domain describe an encounter or proble for the student to cope with, but do not specify what is to be learned. The evaluation is similar to an aesthetic criticism by the instructor.[9]

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[5]Eisner, _op_. _cit_., p. 14.

[6]Eiss and Harbeck, _op_. _cit_., p. 3.

[7]Eisner, _op_. _cit_., p. 15.

[8]Eiss and Harbeck, _op_. _cit_., p. 7.

[9]Eisner, _op_. _cit_., p. 16.

Objectives in both the cognitive and affective domains could be written for any field of study. The remainder of this discussion will be limited to behavioral objectives in the cognitive domain.

Since the early 1900's some educators such as Frederic Birk and Franklin Bobbitt urged teachers to state educational goals in terms of intended behavior changes in the learner. However, they were generally ignored. More recently, Ralph Tyler in 1950 and Bloom in 1956 again stressed writing objectives in this manner. In spite of this, teachers continued to write largely meaningless objectives under orders from the school principal, only to ignore them later. However during the 1960's interest in behavioral objectives swelled because of the potential of precise objectives in improving instruction.[10]

There are several advantages in using precise objectives. First of all, the teacher is able to make better choices regarding what to include in the curriculum, weeding out irrelevant things. Thus the curriculum would consist of the collection of objectives which the teacher is trying to achieve. They also permit a valid assessment of whether or not the students have acquired the desired behaviors, and thus indicate the effectiveness of the instruction.[11]

Secondly, a course with specific behavioral objectives would lend itself to individualized instruction. The teacher could use different

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[10] W. James Popham, "Objectives and Instruction," Instructional Objectives (AERA Monograph Series on Curriculum Evaluation, No. 3. Chicago: Rand McNally & Co., 1969), pp. 32-34.

[11] Howard J. Sullivan, "Objectives, Evaluation, and Improved Learner Achievement," Instructional Objectives (AERA Monograph Series on Curriculum Evaluation, No. 3. Chicago: Rand McNally & Co., 1969), p. 6

methods in teaching the same objectives to students with different aptitudes. This is possible only when a teacher has a clear understandi of course objectives and procedures for measurement.

In connection with individualized instruction, it would be possible to pretest the students. This would prevent wasting valuable time teaching students things they already know. Also the teacher can give assignments which give appropriate practice for the stated objectives and can avoid activities which are essentially irrelevant.

Another important advantage of using precise objectives is that they can be revealed to the students at the beginning of instruction. This practice can promote more relevant study behavior, because too often students try to "psych out" an instructor before an exam and may spend hours learning what is essentially unimportant. If he knew what was expected of him before the exam, the student could use his time much more efficiently.[12]

In spite of these advantages of using behavioral objectives to improve instruction, many teachers find reasons for avoiding them, as Popham explains. One reason given is that since trivial learner behaviors are easiest to write precise objectives for, the really important outcomes of education would be underemphasized. However instead of encouraging trivial outcomes of education, explicitly stating behavioral objectives makes it possible to identify and reject those which are unimportant.

Another reason is that loose general statements of objectives may appear worthwhile to outsiders, but if most goals were stated precisely

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[12] Popham, op. cit., pp. 40-44.

they would be revealed as generally innocuous. This is a potential threat to school people, but unfortunately much of what is going on in schools today is indefensible. If what is being done is trivial, the people who support the educational systems should know it. Educators should be able to defend what they are teaching or revise it.

A third reason for avoiding behavioral objectives is that teachers might be judged on their ability to produce results in learners. This is seen as a threat, especially to the less competent teachers. Howeve this method of evaluation seems better than present methods which often consist of idiosyncratic whims of a visiting supervisor who may feel the teacher is ineffective only because he uses different teaching techniques than the supervisor.

A final reason for avoiding precise behavioral objectives is that they are more difficult to generate than objectives stated in vague ter This is certainly a valid point, but the answer is to reduce the teache load so he has a chance to do a better job.[13]

After weighing all the advantages and disadvantages in using behavioral objectives in a course, a person who decides to use them has a lot of work to do before the course is ready. Instructors in courses in curriculum and instruction who want people to learn about behavioral objectives so they will use them in teaching frequently use Mager's boo <u>Preparing Instructional Objectives</u>, which teaches the learner to ident behavioral objectives written by someone else.[14] However it is important for these people to learn to construct objectives of their

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[13]Popham, <u>op</u>. <u>cit</u>., pp. 46-51.

[14]Sullivan, <u>op</u>. <u>cit</u>., p. 79.

own, so a book such as Walbesser's Constructing Behavioral Objectives could also be used.

The first step in designing a course using behavioral objectives is to identify the terminal objectives, the most difficult tasks the student will be expected to perform upon completion of the course. Then the material the student would have to know in order to complete the terminal tasks must be identified. In other words, the instructor must determine the previously acquired capabilities which would permit the student to complete the given objective under a single set of learning conditions. This process must be progressively applied, beginning with the terminal objective and working backwards until the initial capabilities presumably possessed by all students are reached.[15] This process determines a learning sequence, or hierarchy.

Once the learning sequence has been tentatively set up, then the objectives should be written in specific behavioral terms. According to Walbesser, there are six components which should be included in every behavioral objective:

1) who is to exhibit the behavior
2) what observable performance the learner is expected to exhibit
3) what conditions, objects, and information are given
4) who or what initiates the learner's performance
5) what responses are acceptable
6) any special restrictions on the acceptable response.[16]

These objectives state the observable behavior or action which the learner must perform, so the key word in the objective is the action

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[15] Robert M. Gagne, "Curriculum Research and the Promotion of Learning," Perspectives of Curriculum Evaluation (AERA Monograph Series on Curriculum Evaluation, No. 1. Chicago: Rand McNally & Co., 1967), p

[16] Walbesser, op. cit., p. 24.

verb. In order to be as unambiguous as possible, the number of action verbs should be kept to a minimum and should be defined if there is any doubt as to their interpretation.

Sullivan believes that it is possible to classify nearly all learner behavior related to cognitive tasks in school learning. Accor to him there are six action verbs which could be used to write objecti for any subject area: identify, name, describe, construct, order, anc demonstrate.[17] Walbesser uses these words and adds some others: stat a rule, apply a rule, and distinguish.[18] Other words which are specif related to a certain subject can be added to this list, after defining exactly how they will be used.

Because behavioral objectives describe an observable behavior the student should acquire, evaluation is essential in a course where they are used. The evaluation is derived from a behavioral objective to determine the presence or absense of the desired behavior. This evaluation process is sometimes called an assessment task or a compete measure.[19]

The competency measures should be written by the same person writing the objectives. At least three tasks should be included for each objective, to insure that the student really does possess the desired behavior, and hasn't just made a lucky guess.[20]

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[17]Sullivan, op. cit., p. 75.

[18]Walbesser, op. cit., p. 50.

[19]Ibid., p. 60.

[20]Ibid., p. 61.

There are several restrictions on writing a competency measure. It should assess the same behavior which is described in the objective and should be different from the instructional activity. The action verbs used in the objective and the competency measure should agree. Also, a range of acceptable responses should be specified.[21]

The minimum performance levels should be determined prior to instruction. It is essential to establish high performance levels for certain objectives when mastery of that objective is essential for success on following objectives. If most students have difficulty reaching the minimum performance levels perhaps the instructional material is at fault. In this case the instructional material should be revised, for instance by using more examples and making the material clearer.[22]

Once behavioral objectives have been written and the final hierarchy set up, Gagné describes a way to test the effectiveness of the learning sequence.[23] A test should be made up to determine if the student can or cannot exhibit the desired performance for each objective within the hierarchy. This test should be given to students who have studied the subject at least through the terminal objectives. To determine what material was already learned, the test should also be given to students who have not received instruction in the particular course.

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[22]Sullivan, _op_. _cit_., pp. 82-85.

[23]Gagné, _op_. _cit_., pp. 29-31.

The test should be designed to compare high and low objectives in different areas. To analyze the results, if a large percent get the low objective right and fewer do the high one correctly, then the objectives are probably in the right order. If a large percent do the low one correctly, but only a few are able to do the higher objective, this indicates that more intermediate objectives are needed. Also, if very few get the low one right, but many are able to do the high objective, then the objectives are in the wrong order. This process does not evaluate the curriculum, but merely tells whether a sequence of learning is pedagogically reasonable. It can be used for any curriculum, and lends itself very well to individualized learning. However in an individualized approach certain traditional concepts must be abandoned. The only thing which should be evaluated is whether or not the student has acquired the capability being measured. Degrees of mastery should not be admitted into the measurement. Only after these basic measurements have been made can one ask questions about other kinds of individual differences.[24]

The procedures discussed here have been applied to college algebra. A course has been designed to be taught on an individualized basis. Chapter II is a discussion of suggested classroom procedures. Chapter III includes all of the objectives for the course and a description of each. Chapters IV through VII contain the instructional material for the first four objectives. The material is presented there just as it would be to the students, in the form of a text.

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[24] Gagné, op. cit., pp. 37-38.

CHAPTER II

CLASSROOM PROCEDURES

Following the preceding methods, the author has designed a course in college algebra to be taught on an individualized basis. An individualized approach requires different procedures than those employed in a conventional classroom.

At the beginning of the semester the students should be given a chance to demonstrate their competence in any of the objectives. A pre-measure should be given to all students which covers at least the first 25 objectives. Any material the student can prove he already knows will not have to be repeated later. A student could elect to demonstrate his ability on more than the first 25 objectives, and as soon as he can complete all the objectives satisfactorily, he should receive full credit for the course with a grade of "A".

Upon completion of the pre-measure, each student begins his instruction at the lowest numbered objective which he could not do, and works up the hierarchy at his own pace. He may do the work at home if he wishes, but at least three times a week a classroom should be reserved where he could come to get assistance. The amount of time the room is available would depend on the enrollment. At least one instructor should be available at this time to give special attention to those seeking it. The room should contain reference books to supplement the teacher-designed text. These references should include other algebra texts and books with problems similar to those included in the text.

The students would not be required to attend at any particular time. The only times he would come to the classroom would be to receive assistance or to take a competency measure. He could take as many competency measures in a day as he was capable of. There would be no penalty for not completing a competency measure satisfactorily. The student would simply be required to wait at least one day before making another attempt. He would have to do satisfactory work on it before proceding to the higher objectives.

Using this procedure, a student could go as fast as he wanted, or as slow as he needed to. As soon as he completed all the objectives satisfactorily he would receive an "A" for the course. Other grades would be assigned according to the amount of work completed. Provisions should be made for those who need more time in order to finish. Any student who has made a sincere effort to do the work should be given a chance to complete the course. Anyone with the instructor's permission could elect to receive an incomplete and finish the work the following semester. The student should select a deadline for completion of the work. At that time he would receive the grade corresponding to the amount of work he had done.

CHAPTER III

OBJECTIVES FOR COLLEGE ALGEBRA

Because most of the students who enroll in college algebra are not mathematics majors and only need algebra as a tool in other subjects, the amount of material included in the course was kept to a minimum. The behavioral objectives for the course, along with the criteria for acceptable performance, are included here. They would be given to the students as they appear here, so they would know exactly what to expect.

1. Name the laws of addition and multiplication used in given algebraic statements.

Algebraic statements will be shown to the student and he must name the law which insures that the given statement is true. (Example: $2(x + y) = 2x + 2y$; distributive law.) The statements should contain applications of no more than two rules. When two rules are used, both rules should be named. At least five statements should be given and each law, commutative, associative and distributive, should be used at least once. The student should name all the rules correctly on all statements.

2. Write the sum of two or more polynomials.

The student will be given five problems, each consisting of two or more polynomials in parentheses joined by a plus sign. He must add them by combining all terms with the same literal part into one term.

All five sums must be correct. The problem will be counted wrong if two or more terms with identical literal parts have not been combined into one term.

3. Write the additive inverse of any algebraic expression.

Here an algebraic expression shall mean a polynomial of any degree with less than seven terms; or the sum of fractions with monomials in the denominator. The student will be given five algebraic expressions and must write the additive inverse of each one. Each additive inverse must be correct. If the sign of any term is wrong, then so is the entire problem.

4. Write subtraction problems as addition problems and write the answer.

The student will be given five problems, each containing two or more polynomials in parentheses joined by a plus or minus sign. Those polynomials which are preceded by a minus sign must be transformed to addition by writing the additive inverse of the polynomial. After the entire problem has been written in terms of addition, the student should write the sum as he did in objective 3. Both these things should be done correctly in order for the sum to be correct.

5. Demonstrate the use of the definition of exponents to multiply, divide, and raise monomials to a power.

The student will be given five problems which may combine all three operations. He must write out each monomial according to the definition of exponents (Ex: $a^3b^2 = aaabb$). Then he should write the answer. Each monomial should consist of no more than four variables.

6. Write the product of two or more monomials.

The student will be given five problems, each containing two or more monomials and can find the product either by using the definition as in objective 5 or by using the rule of adding exponents. Each product should be correct. It should be counted wrong if the same variable appea more than once in any individual term.

7. Write the quotient of a monomial divided by another monomial.

The student will be given five problems, each containing a fraction composed of monomials. He must find the quotient by using the definition of exponents or using the rule of subtracting the exponents. It would be counted wrong if the same variable appears in both the numerator and denominator for any particular problem.

8. Write the answer obtained by raising a monomial to a power.

The student will be given five problems, each containing a monomial which should be raised to a power. He must find the answer either by using the definition of exponents or the rule of multiplying the exponent Each answer should be correct.

9. Write the answer obtained by multiplying, dividing, and/or raising a monomial to a power.

The student will be given five problems containing at least two and sometimes all of the three operations. He would have to perform the operations in the correct order and get the correct answer. It would be counted wrong if the same variable appeared more than once in any answer.

10. Write the product of a polynomial and a monomial.

The student will be given five written problems and must multiply a monomial times a polynomial of no more than six terms. The product of the monomial with each term should be simplified as explained in objective 6. Each product must be correct.

11. Write the quotient of a polynomial divided by a monomial.

The student will be given five written problems consisting of polynomials of no more than six terms to be divided by a monomial. Each quotient must be correct, and the quotient of each term and the monomial should be simplified as explained in objective 7.

12. Write the product of two or more polynomials.

The student will be given ten written problems, six of which will be the product of two binomials. These should include ones whose product will be the difference of squares and perfect squares. The others can include at least the sum of two cubes as the product. The total number of terms in both polynomials to be multiplied should not exceed seven. All products should be correct and simplified as in objectives 3 and 6.

13. Write a polynomial in completely factored form.

The student will receive ten polynomials. Five of these should have three terms (including perfect squares), two should have two terms (difference of squares and the sum or difference of cubes), the others should have not more than four terms each. Each polynomial must be factored correctly. In order to be factored completely, the polynomial should be written as the product of other polynomials. The individual factors of the polynomial should be written so that the terms within the factor have no number in common. If it cannot be factored, the student should so indicate.

14. Write the greatest common factor of two or more polynomials.

The student will be given five sets of two to four polynomials, similar to the ones in objective 13. He should write each of them in completely factored form, as in objective 13. Then he should write all the factors which appear in each of the polynomials. The answer can be left in factored form.

15. Write the least common multiple of two or more polynomials.

The student will be given five sets of two to four polynomials, similar to those used in objectives 13 and 14. He should write each of them in completely factored form. All of the factors of each polynomial should appear as often in the least common multiple as the most frequent occurence in any one polynomial. The answer can be left in factored form, and all answers must be correct.

16. Write fractions composed of polynomials in reduced form.

The student will be given five fractions composed of polynomials. First he must write the numerator and denominator in factored form. Then he must find the greatest common factor of the two parts. Since their quotient is one, they may be "cancelled", leaving the answer in reduced form.

17. Write the sum and difference of fractions composed of polynomials

The student will be given five problems composed of adding and subtracting two or more polynomial fractions. He must first find the common denominator (least common multiple), and add the fractions, then reduce them if necessary. All answers must be correct and in reduced form, as in objective 16.

18. Write the multiplicative inverse of algebraic expressions.

The student will be given five algebraic expressions, consisting of three fractions containing polynomials, and two with monomials. He must correctly write the additive inverse, or reciprocal, of each.

19. Write the product and quotient of fractions composed of polynomi

The student will be given five problems composed of multiplicati and division of two or three polynomial fractions. He must first fac them, then perform the indicated operation. All answers must be corr and in reduced form.

20. Write radical expressions so that the smallest number possible i under the radical.

The student will be given ten radical expressions; at least five with numbers, the rest will also include variables. There will be fi square roots, three cube roots, and two with more than degree three. The student must factor each expression into parts which have the indicated root and parts that do not. Then he must find the root of first part and write it outside the radical. All answers must be cor and the number left under the radical must not have a factor with the root.

21. Write the sum of radical expressions.

The student will be given five written problems in which he must add two to six radical expressions as given in objective 20. First he must simplify each expression, and then add them. Each sum must be correc

22. Write the product of radical expressions.

The student will be given five written problems composed of at least one of the following types: product of two racidals, product of a radical times the sum of two or three radicals, or the product of two radical binomials. Each product must be simplified as in objectives 20 and 21.

23. Write fractions containing radicals by rationalizing the denominator

The student will be given five problems consisting of fractions with radicals in the denominator. At least two will have the sum of two radicals in the denominator, the rest will consist of a single term. All but one will be square roots, the other will be a cube root. He must multiply the numerator and denominator by the appropriate number to get a rational number in the denominator, and then simplify the result All answers must be correct.

24. Write the absolute value of numbers.

The student will be given five numbers to write the absolute value of. Each one must be written correctly.

25. Solve linear equations.

The student will be given ten linear equations to solve, with at least three of them containing fractions. He must write a set of equival equations using the rules for solving equations and then write the numeral which is represented by the variable. All solutions must be correct.

26. Check the roots of equations.

The student will be given five equations of any type, and possible roots for each one. He must check each equation by substituting the given numeral and then indicate in writing whether the number is a root of the equation.

27. Solve quadratic equations by factoring.

The student will be given five quadratic equations. He must first set one side of the equation equal to zero, then factor and solve the equations. He must write all solutions for each problem and all must be correct.

28. Solve quadratic equations by the use of the quadratic formula.

The student will be given five quadratic equations and the formula. He must substitute the appropriate numbers into the formula and correctly write all the roots. Equations which are given will have only real roots.

29. Solve equations of degree greater than two which can be factored.

The student will be given five equations which have quadratic and/or linear factors. He must factor them completely and solve them. At least one equation will have a quadratic factor which cannot be factored further. In this case he must also use the quadratic formula. He must correctly write all possible answers for each question.

30. Solve and check equations which have algebraic expressions contained under radicals.

The student will be given five equations with radical expressions; at least one of the equations will contain three such radicals, the others will contain one or two. He must square each side of the equatio at least once and then find all possible solutions. Then he must check each one to be sure it is a root of the original problem. The final answer should have only those solutions which check in the original problem, and all must be correct.

31. Solve equations which contain absolute value.

The student will be given five equations, each with one algebraic expression in absolute value bars. At least two will be quadratic, the others linear. He must correctly write all possible answers for each equation.

32. Solve equations which can be factored and can be written in quadra form through the use of substitution.

The student will be given five equations; in at least two of them the variables will have fractional exponents, and the others will be of degree four or six. The student may solve these by factoring directly or by first making a substitution which will result in a quadratic equa He must correctly write each possible root for each equation.

33. Write equations which describe physical situations and solve

The student will be given five written problems describing physica situations. He must represent the unknown quantity with a variable, an then write the equation which interprets the situation. These equation will all be linear. After the equation is written, the student must solve it as in previous objectives and write the answer.

34. Solve a given formula for a particular variable.

The student will be given five formulas, each one containing at least three variables. He must solve the equation for the indicated variable, writing it in terms of the other variables.

35. Solve linear inequalities.

The student will be given five linear inequalities, with at least two containing fractions. He must write a set of equivalent inequalities, and then write the solution. All must be correct.

36. Construct number line graphs of the solutions of inequalities with only one variable.

The student will take the solutions for the inequalities in objective 35 and construct number line graphs. An open circle at a point will indicate that it is not in the solution, and closed circle at a point indicates it is included. All graphs must be correct.

37. Solve two linear inequalities and find their common solution when it exists.

The student will be given five pairs of linear inequalities similar to the ones given in objective 35. He must solve each separately and then determine the common solution of each pair. He may make graphs of each one if he desires. He must correctly write the common solution of each pair.

38. Solve linear inequalities which contain absolute value.

The student will be given five linear inequalities with algebraic expressions in absolute value bars. He must write the two corresponding inequalities and find their common solution. Each solution must be correctly written.

39. Solve quadratic inequalities which can be factored.

The student will be given four quadratic inequalities which can be factored. He must factor each one and write the possible combinations of inequalities which could produce the given problem. He should have two pairs of inequalities for each problem. He should find the common solution for each pair and then correctly write the solution for each problem.

40. Construct a Cartesian plane and specific points on it.

The student will be given ten ordered pairs of numbers. He must first construct the coordinate axes and then plot the given points. At least two of the points will be on the axes, and at least one in each of the four quadrants. He must correctly plot all ten points.

41. Apply the distance formula to determine the distance between two points on a graph.

The student will be given five pairs of points in the Cartesian plane. One pair will be parallel to one axis, one pair will be in the first quadrant, the others will be in different quadrants. The distance should be found using the distance formula, and each should be correctly written.

42. Apply the distance formula and the definition of a circle to find the equation of a circle when the center and radius can be found.

The student will be given centers of three circles. For one circle the radius will be given, and for the others enough information will be given so that the radius can be found using the distance formula. He should then use the definition of a circle and write the equation of each one correctly.

43. Construct graphs of linear equations.

The student will be given five linear equations and must first make a table of pairs of points which satisfy the equation. Then he must plot the points and draw the line. At least one of the graphs will be parallel to an axis, some will have positive slope, others negative slope. All must be graphed correctly.

44. Construct graphs for equations of degree greater than one.

The student will be given five equations of degree two or greater. He must make a table of values for each one and plot each point on a Cartesian plane. Then he should draw a curved line between these points. Each graph must be correct.

45. Construct a graph of two equations on the same set of axes.

The student will be given five pairs of equations. Each pair is to be graphed on a single set of axes. Two pairs will both be linear, two will contain one linear and one quadratic, and the last one will be two quadratics. All equations must be graphed correctly.

46. Construct graphs for inequalities in the Cartesian plane.

The student will be given five inequalities, at least two of which will be quadratic, the others linear. He must write the equation which corresponds to each one and draw its graph, then shade the appropriate area. Each one must be graphed correctly.

47. Write the value of the determinant of a 2x2 matrix.

The student will be given five 2x2 matricies and must correctly write the determinant of each one. One will contain both variables and numbers. Each must be correct.

48. Write the value of determinants of dimension greater than two.

The student will be given four determinants; three 3-dimensional ones, and one 4-dimensional. The large one will contain at least three zeros. He must evaluate each one by breaking them down to the sum of 2-dimensional determinants. All must be evaluated correctly.

49. Write the value of determinants by first introducing zeros when possible.

The student will be given four determinants to evaluate; three will be 3-dimensional and one 4-dimensional. He should introduce zeros in one row or column where possible. After getting one row or column with all but one element zero, he should then evaluate it. All must be correct.

50. Solve systems of equations using Cramer's Rule.

The student will be given three systems of equations to solve. Two systems will have two equations with two unknowns, the other will have three equations with three unknowns. He must write all necessary determinants and then correctly write the value of each variable.

51. Solve systems of equations using substitution.

The student will be given five systems of two equations each, with two unknowns in each system. Two of the systems will have two linear equations each, one will have two quadratics, and the others will have one linear and one quadratic equation in each. He must solve one of each pair for one unknown and substitute that in the other equation. He must correctly write the value of each variable in each system of equations.

52. Solve systems of equations by eliminating variables.

The student will be given five systems of equations: three with two unknowns and two with three unknowns. He must add these equations to eliminate one or more variables, solve the resulting equation, and find the value of the eliminated variables. He must correctly write the value of each variable in each system of equations.

53. Solve systems of equations using matricies.

The student will be given four systems of equations, two with three unknowns and two with four unknowns. He must solve these using matricies by getting the elements below the main diagonal equal to zero. He must correctly find the solution for each variable in all the systems of equations.

54. Construct graphs to find the solutions to systems of inequalities.

The student will be given five systems of inequalities with two variables in each system. He must graph each system on the same axes and shade the area representing the solution. Three of the systems will be linear, the others will have at least one quadratic. All solutions must be correct.

55. Write the sum of complex numbers.

The student will be given five sets of complex numbers to add, each set will have at least three and at most six complex numbers to add. The student must correctly write each of the five sums. At least half of the complex numbers will be written with a negative number under the radical, and this must be changed to the "i" form before the addition is performed.

56. Write the product of complex numbers in standard form.

The student will be given five sets of complex numbers to multiply. Four sets will contain two numbers, the other will contain three. He must correctly write each product in standard form, a + bi.

57. Write fractions with complex numbers in standard form.

The student will be given five fractions composed of complex numbers. He must write each in standard form by first multiplying the numerator and denominator by the conjugate of the denominator and simplifying each part as in objective 56.

58. Solve equations which have complex roots.

The student will be given five quadratic equations which have complex roots. They must be solved using the quadratic formula and then written correctly in standard form.

Along with these 58 objectives, the student will receive a hierarchy chart as shown in Figure 1. This establishes the order in which the objectives are to be done. The student can be working on more than one part of the hierarchy at a time. The chart contains only the number of each objective. Those which are connected by a line to higher objectives are considered to be prerequisites.

The remaining chapters of this thesis contain the instructional material for the first four objectives. They demonstrate the pattern which is to be used in all the instructional material. They are informally written from an intuitive standpoint, using very few proofs. They are presented here as they would be to the students.

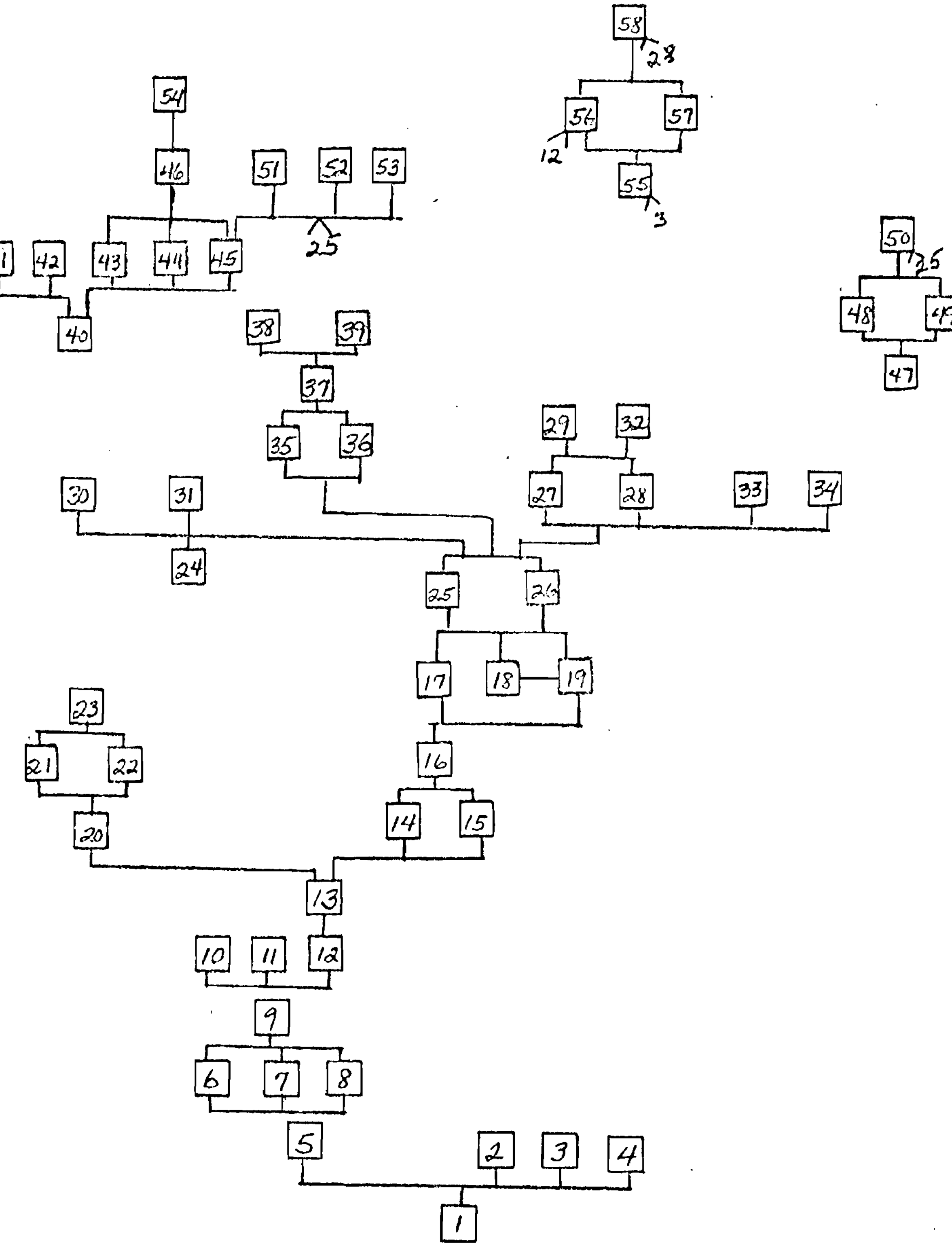

Figure 8. Hierarchy Chart for College Algebra

CHAPTER IV

OBJECTIVE 1

Name the laws of addition and multiplication used in given algebraic statements.

Rationale:

In any field of study there are specialized words which are used extensively. In order to facilitate learning, these basic words need to be learned first. This is certainly true in the case of algebra. There are certain rules which describe the behavior of numbers when they are added or multiplied. You have used these rules since you were first introduced to the addition and multiplication tables, so now you need to associate the rule with its application. In later work in the course there will be statements whose truth may not be obvious. Through the use of the rules, it will be possible to determine the validity of the statements.

Instructional Material:

When you first learned the facts of addition in grade school, you probably noticed that some rules were very similar to others. For instance, 5 + 9 has the same answer as 9 + 5, and 14 + 2 is the same as 2 + 14. This property of numbers, that two numbers can be added in any order, is called the commutative law of addition. The similar rule for multiplication is quite naturally called the commutative law of multiplication. This rule is illistrated by the fact that 2 x 4 is the same as 4 x 2.

At first this law seems so obvious that you may wonder why it is special enough to have its own name. However this property is not true of other operations such as subtraction and division. For instance, is 3 - 1 the same as 1 - 3; or 8/2 the same as 2/8? With subtraction and division it is very important to perform the operations in the given order, whereas the order is not as important in addition or multiplication. Also there are some mathematical systems, which we will not be concerned with in this course, where addition and multiplication are not commutative.

Algebra is in one respect a shorthand for arithmetic. Letters, called variables, are used to represent numbers. The resulting expressions can be treated like numbers. For instance the expression 3a could be 9 if a is 3; or it could be 1 if a = 1/3. What would it be if a is 5? It is possible to work with the expression without knowing its exact numerical value. However the same rules of addition and multiplication can be applied to these variables. Thus 3x + 2y is the same as 2y + 3x; and (4a)(3b) = (3b)(4a), no matter what numbers are represented by the variables. In other words, because the commutative law of addition works for all numbers, and since the variables represent numbers, the law works for the variables too. Stated in general terms, the commutative law says that $a + b = b + a$, and $a \cdot b = b \cdot a$.

Addition and multiplication are binary operations, meaning that they are performed on two numbers at a time. When more than two numbers must be added, they have to be grouped together. Now the question becomes does the way the numbers are grouped together make a difference in the sum? Is the sum of 4 + (5 + 3) the same as the sum of (4 + 5) + 3? Well, 4 + (5 + 3) is the same as 4 + 8 which is 12, and (4 + 5) + 3 is the same as 9 + 3 which is also 12. This is just one xample which could be used to illustrate that the way the numbers are grouped together makes no difference in the sum. The same is true for multiplication. For instance, $(9\cdot3)\cdot4 = 27\cdot4 = 108$, while $9\cdot(3\cdot4) = 9\cdot12 = 108$. Thus $(9\cdot3)\cdot4 = 9\cdot(3\cdot4)$. This property of grouping is given a formal name, associative law. The law for addition in general terms says that $(a + b) + c = a + (b + c)$. The corresponding associative law for multiplication states that $(ab)c = a(bc)$.

> NOTE: In algebra several symbols are used to indicate multiplication. Sometimes the parentheses are used, as in $(a)(4b)$. This could also be written with a dot, $a\cdot4b$, which means the same thing. When two letters are written together, such as xy, this means x times y. Also 3x means 3 times x. Parentheses are also used for grouping. By general agreement, when certain operations appear inside the parentheses, these are performed first. For example, $3(4 + 5)$ means you add the $4 + 5$ before you multiply.

When you have three numbers to be added, you can add the first two together and then add the third, or you can add the first to the sum of the last two. The same procedure applies to multiplication.

Both the preceding rules are applicable when only one operation, addition or multiplication, is involved. When both operations are in one problem, there is a relationship between them. For instance in $3(4 + 7)$, the parentheses mean that 3 is multiplied by the sum of 4 and 7, making it $3\cdot11$ or 33. The parentheses are used as grouping and multiplication symbols. The same answer could be obtained by multiplying 3 times each number and then adding the products. Thus $3(4 + 7) = 3\cdot4 + 3\cdot7 = 12 + 21 = 33$. This property is called the distributive law. Stated in general terms, it says that $a(b + c) = ab + bc$. It can be used when there are more than two numbers in parentheses added together.

All these laws are used quite extensively in algebra. Often you will use them without even realizing it, but you should know what they are in case they are specifically referred to. For easy reference, here they are all together:

Commutative Law of Addition:	$a + b = b + a$
Commutative Law of Multiplication	$a \cdot b = b \cdot a$
Associative Law of Addition:	$(a + b) + c = a + (b + c)$
Associative Law of Multiplication:	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Law:	$a \cdot (b + c) = a \cdot b + a \cdot c$

These laws are all stated in general terms, which means the letters a, b, c can be replaced by other variables or by numbers, and the laws will still be true.

Now let's look and see some applications of these rules to see if you can recognize which one is used. The statement $3m + 4p = 4p + 3m$ is an application of what law? ________ It is the commutative law of addition. Now let's try something more complicated. Which rule is used here: $(2x + 3y) + z = (3y + 2x) + z$. ________ This again is the commutative law of addition because the only part which was changed is the sum of the two numbers inside the parentheses. In the next example, $4k + (2m + 9j) = (2m + 4k) + 9j$, both the commutative and associative laws for addition are used.

Now it is your turn to figure out which laws are used. The answers will be given at the end of the exercises. Remember to write each law and the operation for every statement.

1) $b(2ac) = (2ac)b$ ________

2) $3(x + y + z) = 3x + 3y + 3z$ ________

3) $a + (x + 2p) = (a + x) + 2p$ ________

4) $4(y + 3z) = 12z + 4y$ ________

5) $(3x + 2y)(5z + 7w) = (7w + 5z)(2y + 3x)$ __________

6) $5a(2b \cdot 3c) = (2b \cdot 5a)3c$ __________

7) $((3a + 2b) + 2c) + 4d = (3a + 2b) + (2c + 4d)$ __________

8) $13(x + y) = 13(y + x)$ __________

Throughout this course you will have problems like this to work, and the answers will be included so that you can check yourself. In a situation like this it is important that you work each problem before looking at the answers. Thus you can identify any trouble you might have. Remember, there is no penalty for wrong answers on these problems. When you feel confident about answering these and similar questions, you are ready to take the competency measure. Again there will be no grade on the competency measure, but if you do not answer all the questions correctly, you must take the competency measure again. So it is to your benefit to be honest with yourself when you do the exercises.

Answers:

1) Commutative Law of multiplication

The quantity in parentheses is interchanged with the b.

2) Distributive Law

This law applies here even though there are more than two terms added together inside the parentheses.

3) Associative Law of addition

This is a direct application of the rule.

4) Distributive and commutative laws of addition.

Both these have been used since the y and z terms have been interchanged.

5) Commutative laws of addition and multiplication

The quantities multiplied together have been switched as well as the individual terms within the parentheses.

6) Associative and commutative laws of multiplication

The grouping was changed and then the terms inside the parentheses were commuted.

7) Associative law of addition

Here the (3a + 2b) is treated as a single term and the grouping of the terms has been changed.

8) Commutative law of addition

This is a direct application of the rule - the 13 does not affect the use of the rule.

Hopefully you got all of these sample questions correct. If you did not, did you understand the comments after each answer? If you still do not understand why your answer is wrong, talk it over with the instructor and make sure you understand your mistakes. If necessary, work more problems which the teacher may assign before you take the competency measure. Be sure to seek individual help if you have any questions.

COMPETENCY MEASURE

Objective 1

Each of the five statements below demonstrates the application of the rules of addition and multiplication. You should write the rule or rules in the space provided. Also write the operation involved for the commutative and associative laws. When more than one rule is used, be sure to write all of them. You must write each answer correctly before completing this objective satisfactorily.

1. $2y + (5x + (z + w)) = (2y + 5x) + (z + w)$ __________

2. $5(2x + 3y) = 10x + 15y$ __________

3. $a(3b \cdot c) = a(c \cdot 3b)$ __________

4. $3(7z + 2b) = 6b + 21a$ __________

5. $(2x + y)(3 + z) = (y + 2x)(3 + z)$ __________

CHAPTER V

OBJECTIVE 2

Write the sum of two or more polynomials.

Rationale:

Addition is a very basic operation in mathematics which relates a group of numbers with one number, their sum. You have already learned to add numbers and fractions, and you have undoubtable found this knowledge essential. Addition is just as essential in algebra as the alphabet is in forming words. The main difference from before is that now you need to learn how to add variables, terms with letters in them.

Instructional Material:

What do you get when you add 8 oranges and 3 oranges? Why, 11 orang of course. But what if you add 8 oranges and 3 pickles? Then you still have 8 oranges and 3 pickles!

The preceding example may seem completely unrelated to algebra, but it does illustrate an important point. When you add two quantities which are alike such as $8x + 3x$, you can combine them into one term, $11x$. But when you are adding different quantities such as $8x + 3y$, you must leave them as separate terms. Let's look at this in a more formal way and see how this works.

A term with one or more parts all multiplied together, such as $5xyz$ is called a monomial. The 5 is the coefficient, and the xyz is the literal part. When no coefficient is written, it is understood to be one. Thus xy means 1·xy. An example of the distributive law, as stated in objective one, is $3x(y + z) = 3xy + 3xz$. This can also be used the other way around to state that $3xy + 3xz = 3x(y + z)$.

Now let's use these facts to see how to add monomials. If you want to add $3y + 5y$, the logical answer would seem to be 8y, which is correct. Let's see why. Since there is a y in both terms, $3y + 5y$ is $(3 + 5)y$, which is 8y. It is not necessary to go through this process every time, but this shows how you can apply the simple properti of numbers to add monomials with identical literal parts. All you need to remember when adding monomials like that is that you add the coeffici and leave the literal part the same. So $7y + 3y$ is 10y; $-4z + 8z = 4z$; and $x + (-5x) = -4x$.

What happens when you want to add two monomials with different literal parts, such as $3y + 4z$? The distributive law does not apply here, since you don't have the same literal part in both terms, so there is no way to combine them into one term. This is similar to the earlier example of adding oranges and pickles. You just leave the answer to this as two separate terms added together.

When you have two different monomials added together, this is called a binomial; three terms added together is called a trinomial. The general term which refers to any number of monomials added together is polynomial.

Now that you know how to add monomials, let's extend this to add polynomials. First a simple one: to $2x + 3y$ add $5x + 6y$. Here we have two binomials, both with an x and a y term. Adding the x terms gives a sum of 7x, and the sum of the y terms is 9y. Thus the answer i $7x + 9y$.

Looking at this from a more formal standpoint, this problem is $2x + 3y + 5x + 6y$. Since addition is commutative, you can switch the order of the middle pair, getting then $2x + 5x + 3y + 6y$. Then the first two and the last two pairs can be combined into $7x + 9y$. The answer must be left in this form since the monomials have different literal parts.

When you have a large number of monomials to add together, such as $3x + 2xy + (-5x) + y + 7xy + (-8y) + xy$, you should combine all the terms with identical literal parts into one term. In this example there are two x terms, three xy terms, and two y terms. Since $3x + (-5x) = -2x$; $2xy + 7xy + xy = 10xy$; and $y + (-8y) = -7y$, the entire sum is $-2x + 10xy + (-7y)$. Would it matter if the y term were written first? No, because addition is commutative, you remember. You could get technical about finding the above sum by applying the commutative law many times to get all the terms with identical literal parts together before adding. This is the principle behind adding quantities like this, but it is not necessary to write it all out. You can just take the first term in the line, see if there are any others with the same literal part and add them; then go to the next different term, repeating this process until you have added all like terms. In a problem like this the answer is not complete if you have two or more terms in the same problem with the same literal part. In the preceding example, an answer of $-2x + 9xy + (-7y) + xy$ would not be correct, since two of the terms could be combined further.

Now it is your turn to add polynomials. Again let me remind you not to look at the answers until you finish working all the problems. You are only cheating yourself if you do.

Exercises:

1. $(7ab + 13b) + ab$ __________

2. $(-2x + 3) + (7x + 12)$ __________

3. $(-2mn + 5m) + (-8nm)$ __________

4. $(2y + x + 6) + (3x + 2 + y)$ __________

5. $(-12cd + 7c) + (11cd + 8c)$ __________

6. $(4xz + 2z) + (-3z + 7xy)$ __________

7. $(5y + 2w) + (9w + (-4y))$ __________

8. $(17h + 13k) + (-13h + (-17k))$ __________

9. $(abc + 2b) + (b + cab)$ __________

10. $-14ax + (2bx + 8ax)$ __________

Did you have trouble with 3 or 9? Remember that multiplication is also commutative, so nm is the same as mn. Does that help?

Answers:

1.	$8ab + 13b$	6.	$11xz + (-z)$
2.	$5x + 15$	7.	$y + 11w$
3.	$-10mn + 5m$	8.	$4h + (-4k)$
4.	$3y + 4x + 8$	9.	$2abc + 3b$
5.	$-cd + 15c$	10.	$-6ax + 2bx$

If you answered all these correctly and understand them, you are ready for the competency measure. If not, the teacher can help you individually and refer you to more exercises to get additional practice.

COMPETENCY MEASURE

Objective 2

Find the sums in these five problems. The sum will be counted wrong if there are two or more terms with identical literal parts in the same problem. All five sums must be correct before you satisfactorily complete this unit.

1. $(2ab + 3c + d) + (3d + 2c)$ __________

2. $(4x + 7y + 12z) + (6x + 2z)$ __________

3. $(5b + (-2c)) + (2c + 6b)$ __________

4. $(12xy + 3z) + (8x + 4xy + 6z)$ __________

5. $(-4z + 8y + 7x) + (2x + 3z)$ __________

CHAPTER VI

OBJECTIVE 3

Write the additive inverse of any algebraic expression.

Rationale:

As you just discovered in objective one, the operation of subtraction is not commutative. It is possible to avoid this difficulty by changing a subtraction problem to addition. This process involves finding the additive inverse of the quantities which are subtracted. Thus before we go ahead with the job of converting subtraction problems, let's concentrate on the easier task of writing the additive inverse of algebraic expressions.

Instructional Material:

In our number system there is a number which has some special properties. One of these properties is that when you add this number, zero, to any other number, you get the same number you started with. For instance $3 + 0 = 3$ and $-7 + 0 = -7$. Thus zero is called the additive identity. The additive inverse of any number x, by definition, is the number which must be added to x to get a sum of zero. Thus the additive inverse of 3 is -3, since $-3 + 3 = 0$; and the additive inverse of -10 is 10, since $-10 + 10 = 0$. In general, the additive inverse is opposite in sign to the given number. So if x is the original number, then -x is the additive inverse. Now don't be fooled by the -x: it is not necessarily a negative number. If x happens to be -4, then the additive inverse, or -x, would be 4, a positive number. Also the additive inverse of -x, or -(-x), is x.

Now let's look at some more numbers and their additive inverses.

Number	Additive inverse
11	-11
3y	-3y
-5x	5x
ab	-ab

As you can see, the additive inverses of variables are found in the same way, by taking the term opposite in sign to the original. Again let me caution you about the interpretation of these variables, for instance 3y and its inverse -3y. The natural thing is to assume that the first is positive and the second negative. This is, of course, possible but it is not necessarily true. If y happens to be -5, then 3y would be -15, while -3y would be +15.

Finding the additive inverse of single terms like this is a fairly easy process. It is a little more involved when you have more than one term, but it still is not too difficult. To find the additive inverse of 2x + 3y, just use the general definition of finding the quantity which must be added to this to get zero. Essentially this involves finding the additive inverse of each individual term. Thus the inverse of (2x + 3y) or -(2x + 3y), is (-2x + (-3y)), since the sum of these two quantities is zero.

This brings up a point about notation. You just saw that -(2x + 3y) is -2x + (-3y). When you find the additive inverse of a quantity in parentheses, you must find the additive inverse of each term, and then add each of these together. Let's look at another example. To find the additive inverse of $2x + (-5y)$, or $-(2x + (-5y))$, find the inverse of each term and add, which gives $-2x + 5y$. To check this, take $2x + (-5y)$ and add it to $(-2x) + 5y$. This does equal zero, which shows that $-(-2x + (-5y)) = -2x + 5y$. Similarily, $-(a + (-2b) + 3c) = -a + 2b + (-3c)$; $-(-7y + (-2z)) = 7y + 2z$; and $-(x + y) = -x + (-y)$. You can check all these and show that the sum of each of these quantities and its inverse is zero.

Exercises:

For each of the following expressions write the additive inverse. The answers will be given after the exercises.

	Number	Additive inverse
1.	$1/3$	
2.	$-7z$	
3.	$2x + (-y)$	
4.	$10b + 7c$	
5.	$-15x + 13y + (-c)$	
6.	$-\frac{1}{2}x + (-\frac{1}{4}y)$	
7.	125	
8.	$-6h + 17k + (-3j)$	
9.	$-(a + b)$	
10.	$93m$	

Before you compare your answers with those below, remember that you can check your own work by adding the given quantity to its additive inverse. You should get a sum of zero in each case. Go back now and check your work; you should be able to do it mentally. If you have done this already, then compare your answers with those on the next page.

If you disagree with any of the answers, talk over your problems with the instructor. If you feel you need additional work, the teacher will give you more problems to work on. When you feel confident about your ability, you are ready to take the competency measure.

Answers:

1. $-1/3$
2. $7z$
3. $-2x + y$
4. $-10b + (-7c)$
5. $15x + (-13y) + c$
6. $\frac{1}{2}x + \frac{1}{4}y$
7. -125
8. $6h + (-17k) + 3j$
9. $a + b$
10. $-93m$

COMPETENCY MEASURE

Objective 3

Write the additive inverse of each of the following expressions. The sign of each term must be correct in order for the problem to be counted right. All five problems must be answered correctly before this objective is completed.

Number	Additive inverse
1. $2m + 5p$	
2. $-37x + 3y + (-2z)$	
3. $-y + (-z)$	
4. $2a + 7b$	
5. $-(b + 9c)$	

CHAPTER VII

OBJECTIVE 4

Convert subtraction problems to addition and write the answer.

Rationale:

As we saw in objective one, addition is a commutative operation but subtraction is not. It was certainly easier to add polynomials when we didn't have to worry about what order the terms were in. It would be much easier if there were some way to avoid subtraction all together. Fortunately there is a way, which we will learn about now.

Instructional Material:

There is an interesting relationship which exists between addition and subtraction. The answer to 5 - 3 is 2, while 5 + (-3) is also 2. Also 7 - 4 = 3, while 7 + (-4) is 3 also. By looking at several examples like this, it seems that for every subtraction problem there is a similar addition problem which would have the same answer, and this is exactly the case. This means that a whole new set of rules need not be developed for subtraction. It is possible to convert these to addition problems and use the addition rules we already know. Let's inspect this a little closer to see just how to get the addition problem which has the same answer as the given subtraction problem.

These first examples are very simple, but they will serve to illustrate the procedure. In the first one, 3 is being subtracted from 5, and in the second -3 is added to 5. The 5 stays the same, but the 3 is changed to its additive inverse, -3. In the other example 4 is subtracted from 7, but then its additive inverse, -4, is added to 7. The general rule then is to take the number being subtracted, and instead add its additive inverse. Subtraction is defined this way in terms of addition, and the definition is stated algebraicly as follows: $a - b = a + (-b)$.

Applying this definition, $12y - 5y$ becomes $12y + (-5y)$ which is $7y$. For a problem like this which is so easy it probably seems unnecessary to change it to addition first. There are certainly many problems that would fit into this category. This process of converting to addition has the most value for more difficult problems.

How would you work $-12y - 5y$, or $12y - (-5y)$? These are a little more difficult, but by converting to addition, or adding the additive inverse, they become $-12y + (-5y) = -17y$, and $12y + 5y = 17y$. These addition problems are generally easier for most people to work, but again these subtraction problems could be worked correctly without converting to addition. The same rules apply in checking a subtraction problem that you learned in grade school: $a - b = c$ means $c + b = a$, or $9 - 5 = 4$ means $4 + 5 = 9$. Thus $-12y - 5y = -17y$ because $-17y + 5y = -12y$, and $12y - (-5y) = 17y$ because $17y + (-5y) = 12y$.

Subtraction remains a fairly simple operation as long as you are just subtracting monomials. But what happens when you start subtracting polynomials: for instance $(5x + 2y) - (2x + 3y)$. Here the entire quantity $2x + 3y$ is being subtracted. Using the definition of subtraction, this means that the additive inverse of $2x + 3y$ must be added to the first binomial. But what is the additive inverse of $2x + 3y$? It is

$-2x + (-3y)$. Thus our subtraction problem becomes $5x + 2y + (-2x) + (-3y)$ which is $3x + (-y)$. This answer could also be written as $3x - y$, which means the same thing.

How was this problem different from the following: $5x + 2y - 2x + 3$ In the first one, the entire quantity $2x + 3y$ in parentheses is subtracted The second example becomes $5x + 2y + (-2x) + 3y$ which is $3x + 5y$. So you can see that parentheses do make a difference in a problem. When a quantity inside parentheses is preceded by a minus sign, this means the entire quantity is subtracted. When no parentheses are used, the minus sign only goes with the term immediately following it.

As you have seen, the minus sign is used two ways. It can mean the additive inverse, as in $-4z$, or it can mean subtraction, as in $9z - 4z$. This need not cause any confusion however. $9z - 4z$ really means $9z + (-4z)$, but quite often the plus sign is omitted, but you get the same answer anyway. Hereafter when you see problems such as $3x - 2x - 5x$, this means $3x + (-2x) + (-5x) = -4x$, but many times the plus sign is omitted to conserve space. You should get in the habit of supplying the missing plus signs, at least mentally. In doing this you are essentially converting subtraction to addition.

Let's look at some more examples of subtracting polynomials. In the problem $(5y + 7z + 2) - (6y + 2z + 8)$, the quantity $6y + 2z + 8$ is being subtracted, so the additive inverse of that quantity should be added to the first. Thus $(5y + 7z + 2) - (6h + 2z + 8) = 5y + 7z + 2 + (-6y) + (-2z) + (-8)$ which is $-y + 5z + (-6)$ or $-y + 5z - 6$. The additive inverse of a sum of numbers turns out to be the sum of the additive inverses of the numbers.

In this problem, $(-2a + 3b) - (4a - 2b)$ the quantity $4a - 2b$, or $4a + (-2b)$ is subtracted, so the additive inverse, $-4a + 2b$ should be added. Thus $(-2a + 3b) - (4a - 2b) = -2a + 3b + (-4a) + 2b$, or $-6a + 5b$. If you will notice, when you convert to addition or remove the parentheses, you end up adding quantities which are opposite in sign to the quantities inside the parentheses. Thus $(-3xy - 4z + y) - (3y + xy - 5z)$ becomes $-3xy + (-4z) + y + (-3y) + (-xy) + 5z$ which is $-4xy + z - 2y$. When the second set of parentheses was removed, the sign of every term inside was changed. This is a very important point to remember. Often in problems like this with several terms inside the parentheses, a common mistake is to forget to change the sign of all the terms inside. (The term "removing the parentheses" implies converting from subtraction to addition.)

A problem with more than two quantities involved, such as $3x + 7y - (2y - 3x) + 2x - 7y$, the entire problem should first be written in terms of addition, getting $3x + 7y + (-2y) + 3x + 2x + (-7y)$, which is $8x + (-2y)$ or $8x - 2y$. I think you can see that as the problems get more involved, it is indeed easier to change everything to addition before going ahead to find the answer.

Now you are ready to apply what you have learned about subtraction. Write your answers to the following exercises, and discuss any problems with your instructor.

Exercises:

1. $(4y + 7z) - (2y + 4z)$
2. $3y - 10x + 18y - 12z + 3x$
3. $6x - (5 - 4x)$
4. $12y + 5z - (3x - 8z)$
5. $(7xy + 2z) - (3z - 2xy)$
6. $(-31x - 7y + 4) - (2x + 8y - 2)$
7. $3x - (8 - 7x)$
8. $14a - 6b + 44c - 81b - 12c$

COMPETENCY MEASURE

Objective 4

Write each of these five problems in terms of addition, then write the answer. The answer will be counted wrong if there are two or more terms with identical literal parts in the same problem. All five answers must be correct before you satisfactorily complete this unit.

1. $(3xy + 2z) - (4 - 3xy + 2z)$

2. $(4a - 5b + 7) - (2a + 9b - 3)$

3. $(6x - 8y) + (2x + 3y) - (12y - 4x)$

4. $(6 - x) - (4 - x)$

5. $(5a - 7b + 3c - 9d) + (4a - 2c) - (5b + 8d)$

CHAPTER VIII

SUMMARY

The students who enroll in college algebra have a varied background in mathematics. A course taught on an individualized basis seemed the best way to meet their needs. The author's purpose in this thesis was to design such a course.

Behavioral objectives were written for the course because they are ideally suited for an individualized course. The goals of the course were stated clearly so the student would know exactly what was expected of him. A hierarchy chart was designed so that the student could plan his own course of study, going at his own rate.

Also included in this thesis is the instructional material for the first four objectives, to illustrate how the remainder of the text would be written. The style is very informal so that the student can understand it and do most of the work on his own.

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