

THE CONSTRUCTION AND STANDARDIZATION OF A
TEST IN PLANE GEOMETRY

A THESIS

SUBMITTED TO THE DEPARTMENT OF
PSYCHOLOGY AND THE GRADUATE COUNCIL OF THE KANSAS STATE
TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

By

IDA SARAH BECKER

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Approved for the Major Department

H. E. Schramm

Approved for the Graduate Council

Edwin Brown

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CHAPTER I

PURPOSE AND PROCEDURE

In the school year of 1932-33 the writer had the privilege of constructing the plane geometry tests for the two Nation-wide Every Pupil Scholarship Tests annually sponsored by the Bureau of Educational Measurements of K.S.W.C., Emporia. These tests were used in a large number of schools over the country.

A study of the results available from the tests revealed the fact that there is a need for a good standardized test in the field of plane geometry. As the writer felt that the provision of such a test would prove a valuable and interesting project, it was determined that she should provide tests which might be standardized and which might be used in the testing programs for the two succeeding years.

In the fall of 1933, two tests covering the first four months of work were given to 200 students in the high schools at Hutchinson and Topeka. After being equalized on the basis of student responses, these two tests became Forms A and B of Test I.

This study consists of a description of the details of standardizing the tests and an analysis of the results obtained from the use of the tests.

Schools Taking the Tests

Although Form A of both Tests I and II was used by a great many schools in the Nation-wide Testing Program, both forms of both tests were given all pupils enrolled in plane geometry classes in a limited number of schools. The purpose of this procedure was to make an intensive study of the comparability of the corresponding forms and to refine the instruments. In Table I

are listed the schools used for this purpose and the number of students taking each form of the test.

TABLE I
NUMBER OF STUDENTS TAKING EACH TEST
IN THE VARIOUS SCHOOLS

Schools	Test I		Test II	
	A	B	A	B
Abilene	99	97	--	--
Americus	14	13	13	13
Bennington	22	22	20	19
Ellsworth	45	43	40	38
Frankfort	58	58	58	58
Hill City	--	--	24	26
Herington	48	48	45	45
LaHarpe	20	20	16	19
Leavenworth, Immaculata	74	69	69	65
Madison	28	28	24	26
Miller	8	8	9	10
Salina	--	--	101	101
Winfield	104	102	103	104
Total taking test	520	508	522	524

Read table thus: In the Abilene High School 99 students took Test I, Form A; 97 students took Test I, Form B.

Scope of the Material

This series of tests consists of two groups, designated in this study as Test I and Test II. Each group is made up of two equivalent forms, Form A and Form B. For each test the two forms are equal in respect to number of parts, number of items per part, and type of questions. The content of the corresponding parts of Form A and Form B of each test is equivalent.

The subject matter covered by the tests is so arranged that Test I may be given upon the completion of four months of work and Test II, upon the completion of approximately eight months of work. The content of the test conforms to the subject matter as outlined by the Kansas State Course of Study, the recommendations of the Report of the National Committee on Mathematics Requirements, and the College Entrance Examination Board.

Test I covers the following material: definitions; symbols and abbreviations; fundamental facts; constructions; rectilinear figures, such as angles, triangles, and parallel lines; quadrilaterals and polygons; and circles up to and including chords, arcs, and central angles.

Test II covers the following material: definitions; symbols; abbreviations; fundamental facts; rectilinear figures, such as angles, triangles, and parallel lines; quadrilaterals and polygons; circles; areas; ratio and proportion; loci; problems; fundamental construction for fourth and mean proportions; and other construction problems.

Types of Questions

In Table II are summarized the number and types of test items used in the two forms of Test I and Test II.

TABLE II
NUMBER AND TYPES OF TEST ITEMS IN EACH TEST

Type of item	Test I		Test II	
	A	B	A	B
Completion	33	33	30	30
Matching	10	10	--	--
Problems	9	9	15(2)*	15(2)*
Proof	34	34	30	30
Constructions	6(5)*	6(5)*	6(5)*	6(5)*
Total points possible	116	116	122	122

Read table thus: In Test I, Forms A and B, there were 33 items in the completion part. In Test II, Form A and B, there were 30 items in the completion part.

*The problems of Test II, Forms A and B, each counted two points. Constructions in all forms were weighted, each correct construction counting 5 points.

It will be observed that in Test I five types of questions are used, namely: completion, matching, problems, proof, and constructions. The completion part, Part I, consists of 33 items covering definitions and theorems, Part II, also covering definitions and theorems, consists of ten matching questions with 23 choices. The large number of choices tends to control the problem of guessing. Part III is made up of nine numerical problems; Part IV, of three theorems in which the student has to give the statements of the proof and their reasons; and Part V, of six constructions in which the student is to use compass and straight edge.

Test II consists of the following parts: completion, problems, proof, and constructions. The completion part, Part I, contains 30 items covering definitions and theorems; Part II, 15 applied problems; Part III, three theorems for which the student must give the statements of the proof and their reasons; and Part IV, six constructions in which compass and straight edge are used.

The several tests and the scoring keys provided for them follow.

TEST I
FORM A

Time: 40 minutes.

Published by
Bureau of Educational Measurements
Kansas State Teachers College, Emporia

Possible score 116

PLANE GEOMETRY

By Ida S. Becker, Americus, Kansas

STUDENT'S

SCORE

Name Age Grade

School Teacher

Town State Date

PART I

(Possible score, 33)

Score.....

DIRECTIONS: In each of the following statements a word, phrase, or number has been omitted where the stars (****) are placed. Write the appropriate word, phrase, or number on the line below the statement.

1. An angle formed by two perpendicular lines is a ****.
.....
2. An angle whose size is between a zero angle and a right angle is called an ****.
.....
3. Two angles whose sum is equal to a straight angle are called ****.
.....
4. A theorem which follows immediately as a consequence of some other theorem is called a ****.
.....
5. Complements of equal angles are ****.
.....
6. A triangle with no two sides equal is a ****.
.....
7. In a right triangle the side opposite the right angle is called the ****.
.....
8. Two line segments or two angles that may be made to coincide are ****.
.....
9. A line intersecting each of two or more lines is called a ****.
.....
10. The line from a vertex of a triangle to the midpoint of the opposite side is a ****.
.....
11. The length of the broken line which forms a polygon is called its ****.
.....
12. If the angles of a polygon are equal respectively to the angles of another polygon, taken in the same order, the polygons are said to be ****.
.....
13. The sum of the two acute angles of any right triangle is ****.
.....
14. Two angles whose sides are respectively parallel are ****.
.....
15. The sum of the interior angles of a polygon of n sides is ****.
.....
16. The locus of all points equidistant from the sides of an angle lies on the **** of the angle.
.....
17. If a line moves except along itself it is said to generate a ****.
.....
18. To cut into two equal parts means to ****.
.....
19. The bisectors of two complementary adjacent angles form an angle of **** degrees.
.....
20. In any triangle the angle opposite the base is called the **** angle.
.....
21. If the three altitudes of a triangle are equal, the triangle is ****.
.....
22. A quadrilateral having two and only two of its sides parallel is a (an) ****.
.....
23. The line which bisects two sides of a triangle is **** to the third side and equals one half of it.
.....
24. The size of the smaller angle formed by the hands of a clock at 2 o'clock is ****.
.....
25. If $BC = \frac{1}{2}AB$ and $AB = XY$, then $\frac{1}{2}XY =$ ****, in terms of BC .
.....

26. The supplement of a 38° angle is ****.

.....

27. An angle whose complement is 68° more than the angle itself equals ****.

.....

28. A central angle of 80° intercepts an arc of ****.

.....

29. The total number of degrees in the interior angles of a hexagon is ****.

.....

30. The number of sides of an equiangular polygon, each of whose exterior angles equals 20° , is ****.

.....

31. If four angles of a pentagon are 95° , 90° , 100° , and 110° , the fifth one is ****.

.....

32. If the median of a trapezoid is 13 and the lower base is 18, the upper base is ****.

.....

33. If $a > b$ and $b = c$, then a **** c .

.....

PART II

(Possible score, 10)

Score

DIRECTIONS: Read each of the following statements and think what word or words complete it. Then find the word or phrase in Column II and write its number in the parenthesis before the statement. An answer may be used more than once.

Column I.

Column II.

- | | |
|--|---|
| <p>() 1. Two straight lines perpendicular to the same line are</p> <p>() 2. Two congruent figures can be made to</p> <p>() 3. The interior angles of a rectangle are</p> <p>() 4. If equals are subtracted from equals the differences are</p> <p>() 5. An exterior angle of a triangle is greater than either non-adjacent</p> <p>() 6. If two adjacent angles have their exterior sides in the same straight line they are</p> <p>() 7. In any triangle at least two of the angles are</p> <p>() 8. If two consecutive angles of a quadrilateral are right angles, the bisectors of the other two angles are</p> <p>() 9. A plane figure with N sides is a</p> <p>() 10. A line which touches a circle in only one place is a</p> | <p>1. acute</p> <p>2. area</p> <p>3. chord</p> <p>4. circle</p> <p>5. coincide</p> <p>6. complementary</p> <p>7. equal</p> <p>8. exterior angle</p> <p>9. interior angle</p> <p>10. isosceles</p> <p>11. obtuse</p> <p>12. parallel</p> <p>13. perimeter</p> <p>14. perpendicular</p> <p>15. polygon</p> <p>16. proof</p> <p>17. right angles</p> <p>18. rhombus</p> <p>19. secant</p> <p>20. supplementary</p> <p>21. tangent</p> <p>22. trapezoid</p> <p>23. vertical angle</p> |
|--|---|

PART III

(Possible score, 9)

Score

DIRECTIONS: In each of the following problems compute the answer to each question and write it on the dotted line.

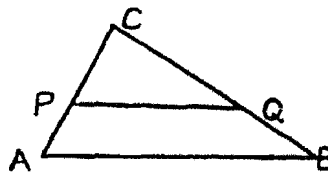


Fig. 1

I. (Figure 1)

$PQ \parallel AB$

$\angle B = \frac{1}{2} \angle A$

$\angle PQB = 147^\circ$

1. $\angle PQC = \dots\dots\dots^\circ$

2. $\angle CAB = \dots\dots\dots^\circ$

3. $\angle ACB = \dots\dots\dots^\circ$

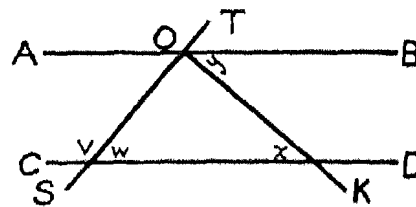


Fig. 2

II. (Figure 2)

$AB \parallel CD$

$KS \perp ST$

$\angle v = 128^\circ$

4. $\angle x = \dots\dots\dots^\circ$

5. $\angle y = \dots\dots\dots^\circ$

6. $\angle w = \dots\dots\dots^\circ$

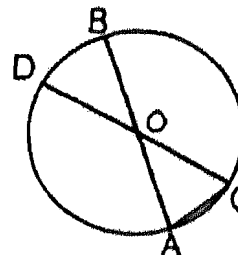


Fig. 3

III. (Figure 3)

O is the center of circle

$\text{Arc } AC = 40^\circ$

7. $\angle BOB = \dots\dots\dots^\circ$

8. $\angle OAC = \dots\dots\dots^\circ$

9. $\angle BOC = \dots\dots\dots^\circ$

PART IV

(Possible score, 34)

Score

DIRECTIONS: In each of the following problems study the figure and the statements of what is given and what is to be proved. Write the steps of the proof on the lines left for this purpose under "Proof." Next find the reason for each step of your proof in the accompanying "List of Reasons," and write its number in the parenthesis following the step. There should be as many steps in a correct proof as there are lines left for them.

List of Reasons

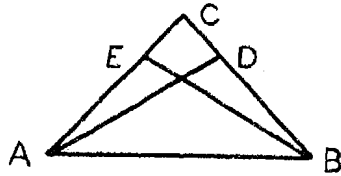


Fig. 4

GIVEN:

$AC=BC$

$BD=AE$

TO PROVE:

$\triangle ABE \cong \triangle ABD$

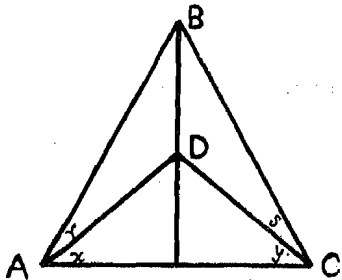


Fig. 5

GIVEN:

$AB=BC$

$AD=DC$

TO PROVE:

$\angle r = \angle s$

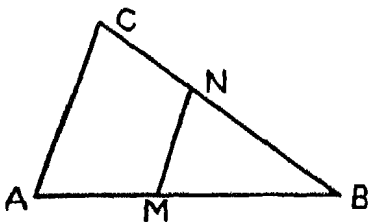


Fig. 6

GIVEN:

$AB=BC$

$MN \parallel AC$

TO PROVE:

$MB=NB$

Proof	Reasons
Example (a). $AC=BC$	(a) (1)
1.	1. ()
2.	2. ()
3.	3. ()
4.	4. ()
5.	5. ()
6.	6. ()
7.	7. ()
8.	8. ()
9.	9. ()
10.	10. ()
11.	11. ()
12.	12. ()
13.	13. ()
14.	14. ()
15.	15. ()
16.	16. ()
17.	17. ()

1. Given.
2. Identical.
3. Two triangles are congruent if three sides of one are equal respectively to three sides of the other.
4. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.
5. Two triangles are similar if three angles of one are equal respectively to three angles of the other.
6. Two triangles are congruent if two angles and the included side of one are equal respectively to two angles and the included side of the other.
7. If two parallel lines are cut by a transversal, the alternate interior angles are equal.
8. If two parallel lines are cut by a transversal the corresponding (exterior-interior) angles are equal.
9. If two lines are cut by a transversal making the alternate interior angles equal, the lines are parallel.
10. If two sides of a triangle are equal, the angles opposite these sides are equal.
11. If two angles of a triangle are equal, the sides opposite these angles are equal.
12. Equals may be substituted for equals.
13. If an angle is divided into two equal parts by a line, the line is called the bisector of the angle.
14. Quantities which are equal to the same quantity, or to equal quantities, are equal to each other.
15. Corresponding parts of congruent triangles are equal.
16. If equals are subtracted from equals, the differences are equal.

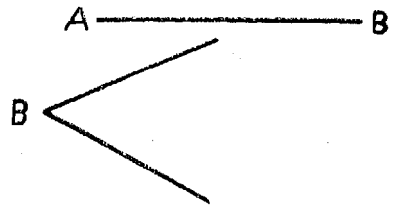
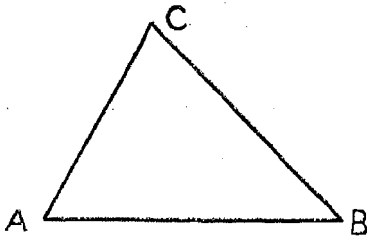
PART V

(Possible score, 30)

Score.....

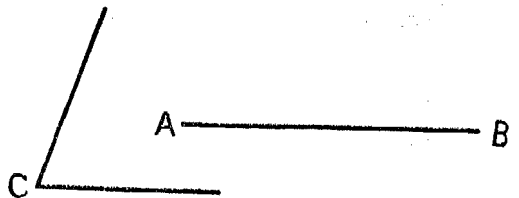
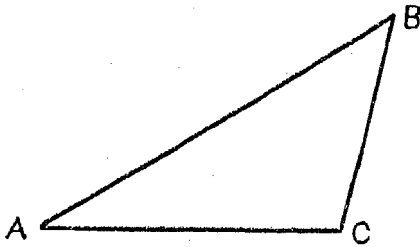
DIRECTIONS: In these exercises you are to make certain formal constructions. Compass and straight-edge are to be used. All necessary construction lines should appear in the completed figure.

1. Find a point O equally distant from the three vertices of the triangle ABC.
2. Construct the altitude BE to side AC of the triangle ABC.
3. On line AB construct $\triangle ABC$ in which $\angle A$ is as given and $\angle B = \frac{1}{2} \angle A$.
4. Construct an isosceles triangle ABC in which $\angle B$ vertex angle and AB is a side.
5. On line AB construct a right triangle ABC when $\angle C$ and line AB are as given.
6. At P on line MN construct an angle OPN which will be 135° .



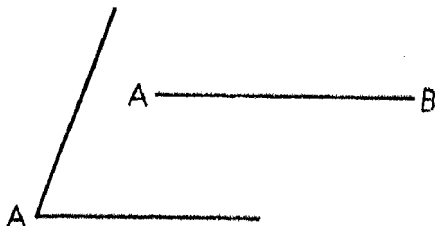
2. Construct the altitude BE to side AC of the triangle ABC.

5. On line AB construct a right triangle ABC when $\angle C$ and line AB are as given.



3. On line AB construct $\triangle ABC$ in which $\angle A$ is as given and $\angle B = \frac{1}{2} \angle A$.

6. At P on line MN construct an angle OPN which will be 135° .



GEOMETRY TEST

Test I

Form B

PART I

DIRECTIONS: In each of the following statements a word, phrase, or number has been omitted where the stars (***) are placed, or a question has been asked. Write the appropriate word, phrase, number, or answer on the line at the left of the statement.

- Adjacent angles 1. Two angles which have a common vertex and a common side between them are called ***.
- Obtuse angle 2. An angle whose size is between a right angle and a straight angle is an (a) ***.
- Complementary 3. Two angles whose sum is equal to a right angle are ***.
- Theorem 4. A statement whose truth admits of a proof is called a (an) ***.
- Equal 5. Supplements of equal angles are ***.
- Isosceles triangle 6. A triangle that has any two of its sides equal is called (a) an ***.
- Legs 7. In a right triangle the two sides of the right angle are called ***.
- Congruent 8. Two closed figures which can be made to coincide are said to be ***.
- Parallel lines 9. Lines that lie in the same plane and that cannot meet however far produced are called ***.
- Altitude 10. The line from a vertex of a triangle perpendicular to the opposite side is called (a) an ***.
- Diagonal 11. The line joining any two non-consecutive vertices of a polygon is (a) an ***.
- Congruent 12. Two polygons which are mutually equiangular and mutually equilateral can be made to coincide and are said to be ***.
- 180° 13. The sum of the three angles of any triangle is equal to ***.
- Equal or supplementary 14. Two angles whose sides are respectively perpendicular to each other are ***.

- 360° 15. The sum of the exterior angles of a polygon formed by producing the sides in succession is ***.
- Perpendicular bisector 16. The locus of all points equidistant from the extremities of a line is the ***.
- line 17. A moving point is said to generate a (an) ***.
- Bisector 18. If a line divides an angle or another line into two equal parts it is called a (an) ***.
- 90° 19. The bisectors of two supplementary adjacent angles form an angle of ***.
- Hypotenuse 20. In any right triangle the side opposite the right angle is called the ***.
- Isosceles 21. If only two altitudes of a triangle are equal the triangle is ***.
- Quadrilateral 22. A polygon formed by four straight lines is a (an) ***.
- Medians 23. The *** of a triangle meet in a point which is two-thirds the distance from any vertex to the mid-point of the opposite side.
- 120° 24. The size of the smaller angle formed by the hands of a clock at 4 p.m. is ***.
- $\frac{1}{2}BC$ 25. If $BC = \frac{1}{3} AB$ and $AB = 2 XY$, then $\frac{1}{6} XY =$ *** in terms of BC .
- 43° 26. The complement of a 47° angle is ***.
- 55° 27. An angle whose supplement is 70° more than the angle itself equals ***.
- 60° 28. A central angle of 60 degrees intercepts an arc of ***.
- $65^\circ - 35^\circ - 80^\circ$ 29. If the sum and the difference of two angles of a triangle are respectively 100° and 30° , the three angles are ***.
- 540° 30. The total number of degrees in the interior angles of a pentagon is ***.
- 40 31. An equiangular polygon, each of whose exterior angles equals 9° has *** sides.
- 11 32. If the upper and lower bases of a trapezoid equal 15 and 7 respectively, the length of the median is ***.
- 160° 33. Two parallel lines are cut by a transversal. If one of the two interior angles on the same side of this transversal is eight times the other, the number of degrees in the larger angle is ***.

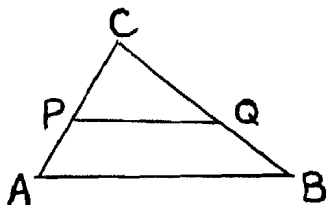
PART II

DIRECTIONS: Read each of the following statements and think what word or words complete it. Then find the word or phrase in the list of answers and write its number in the parenthesis before the statement. An answer may be used more than once.

- | | | |
|------|--|--------------------|
| (19) | 1. Perpendicular makes not only equal adjacent angles with the line but also | 1. Acute |
| (17) | 2. In a theorem the line of argument leading to a conclusion is called the | 2. Area |
| (6) | 3. Alternate interior angles of parallel lines are | 3. Chord |
| (14) | 4. The opposite sides of a parallelogram are equal and | 4. Conclusion |
| (9) | 5. The number of sides of a quadrilateral is | 5. Complementary |
| (2) | 6. The number of square units in a closed plane figure is called the | 6. Equal |
| (6) | 7. If equals are divided by equals, the quotients are | 7. Exterior angle |
| (20) | 8. A straight line which intersects a circle in two points is called a | 8. Five |
| (13) | 9. The supplement of the complement of an acute angle is always | 9. Four |
| (11) | 10. If the non-parallel sides of a trapezoid are equal, the trapezoid is | 10. Given |
| | | 11. Isosceles |
| | | 12. Interior angle |
| | | 13. Obtuse |
| | | 14. Parallel |
| | | 15. Perimeter |
| | | 16. Perpendicular |
| | | 17. Proof |
| | | 18. Unequal |
| | | 19. Right angles |
| | | 20. Secant |
| | | 21. Supplementary |
| | | 22. Tangent |
| | | 23. Trapezoid |

PART III

DIRECTIONS: In each of the following numerical problems you are to compute the answer to the question and write it on the dotted line.



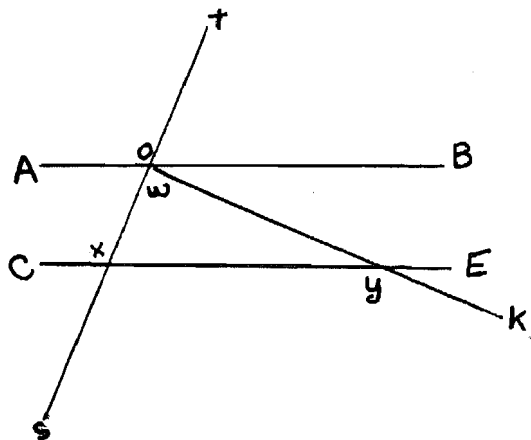
I. (Figure 1)

$PQ \parallel AB$

$$\angle APQ = 118^\circ$$

$$\angle PQB = 146^\circ$$

1. $\angle C = \underline{\quad 84 \quad}^\circ$
2. $\angle A = \underline{\quad 62 \quad}^\circ$
3. $\angle B = \underline{\quad 34 \quad}^\circ$



II. (Figure 2)

$AB \parallel CE$

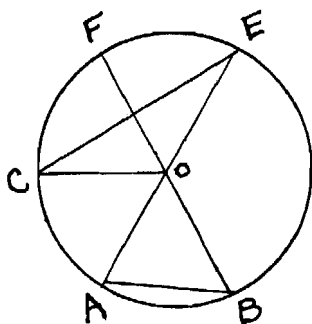
$ok \perp st$

$\angle toB = 28^\circ$

4. $\angle x = \underline{\underline{152}}^\circ$

5. $\angle y = \underline{\underline{118}}^\circ$

6. $\angle w = \underline{\underline{90}}^\circ$



III. (Figure 3)

o is the center of \odot

$FB \perp CE$

$\angle CEO = 28^\circ$

7. $\angle OBA = \underline{\underline{59}}^\circ$

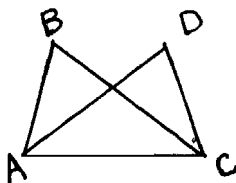
8. $\angle AOB = \underline{\underline{62}}^\circ$

9. $\angle EOB = \underline{\underline{118}}^\circ$

PART IV

DIRECTIONS: In each of the following problems study the figure and the statements of what is given and what is to be proved. Write the steps of the proof on the lines left for this purpose under "proof." Next find the reason for each step of your proof in the accompanying list of "reasons." Write its number in the parenthesis following the step. There should be as many steps in a correct proof as there are lines left for them.

1.



Given: $AB = DC$
 $\angle BAC = \angle DCA$

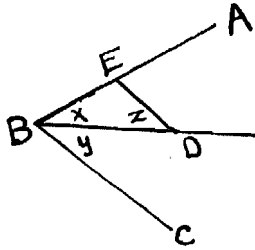
To prove: $\triangle ABC \cong \triangle ADC$

Proof

Reasons

- | | |
|--|----------|
| 1. $AB = DC$ | 1. (1) |
| 2. $AG = AC$ | 2. (2) |
| 3. $\angle BAC = \angle DCA$ | 3. (1) |
| 4. $\triangle ABC \cong \triangle ADC$ | 4. (4) |

2.

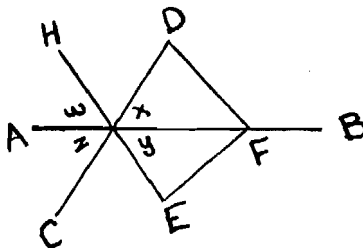


Given: $BE = ED$
 $ED \parallel BC$

To prove: $\angle ABC$ is bisected by BD

- | | |
|---|----------|
| 5. <u>$BE = ED$</u> | 5. (1) |
| 6. <u>$\angle x = \angle z$</u> | 6. (8) |
| 7. <u>$ED \parallel BC$</u> | 7. (1) |
| 8. <u>$\angle z = \angle y$</u> | 8. (6) |
| 9. <u>$\angle x = \angle y$</u> | 9. (13) |
| 10. <u>$\angle ABC$ is bisected by BD</u> | 10. (12) |

3.



Given: $DO = OE$
 $\angle y = \angle z$

To prove: $DF = EF$

- | | |
|---|----------------|
| 11. <u>$DO = OE$</u> | 11. (1) |
| 12. <u>$OB = OB$</u> | 12. (2) |
| 13. <u>$\angle y = \angle z$</u> | 13. (1) |
| 14. <u>$\angle z = \angle x$</u> | 14. (15) |
| 15. <u>$\angle x = \angle y$</u> | 15. (11) or 13 |
| 16. <u>$\triangle DOF \cong \triangle OEF$</u> | 16. (4) |
| 17. <u>$DF = EF$</u> | 17. (14) |

Reasons for Part IV

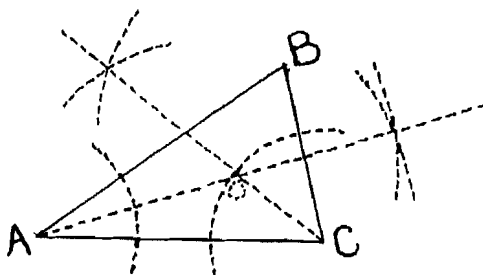
1. Given.
2. Identical.

3. Two triangles are congruent if three sides of one are equal respectively to three sides of the other.
4. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.
5. Two triangles are congruent if two angles and the included side are equal respectively to two angles and the included side of the other.
6. If two parallel lines are cut by a transversal, the alternate interior angles are equal.
7. If two parallel lines are cut by a transversal the exterior-interior angles are equal.
8. If two sides of a triangle are equal, the angles opposite these sides are equal.
9. If two angles of a triangle are equal, the sides opposite these angles are equal.
10. If equals are subtracted from equals, the differences are equal.
11. Equals may be substituted for equals.
12. If an angle is divided into two equal parts by a line, the line is called the bisector of the angle.
13. Quantities which are equal to the same quantities, or to equal quantities, are equal to each other.
14. Corresponding parts of congruent triangles are equal.
15. Vertical angles are equal.
16. Complements of equal angles are equal.

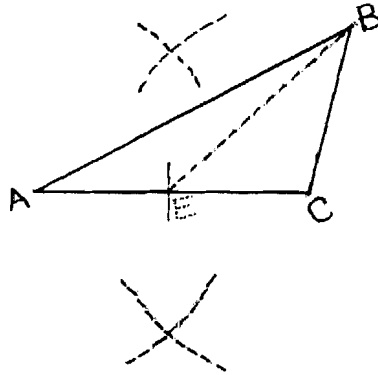
PART V

DIRECTIONS: In these exercises, you are to make certain formal constructions. Compass and straight-edge alone are to be used. All necessary construction lines should appear in the completed figure.

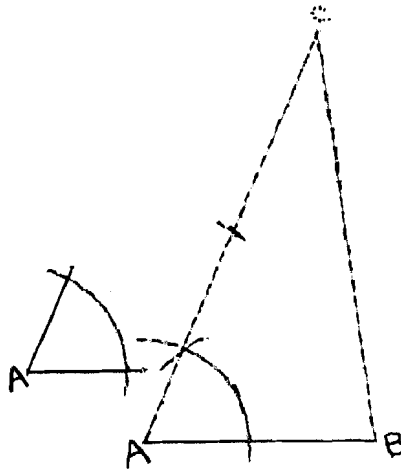
1. Find a point O equally distant from the three sides of triangle ABC.



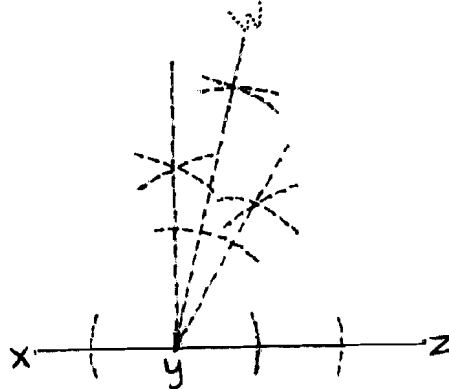
2. Construct the median BE to side AC of triangle ABC



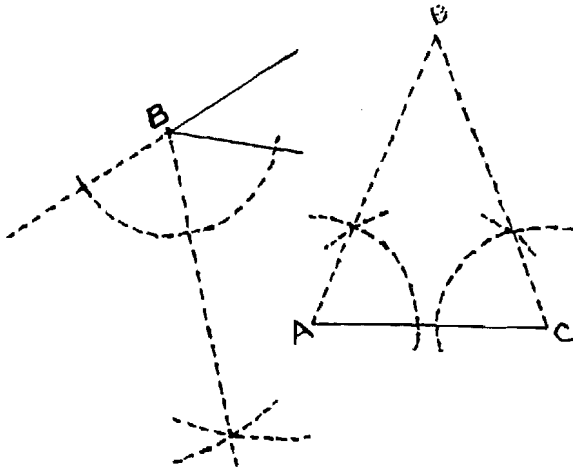
3. Construct a triangle ABC in which $AC = 2AB$ and $\angle A$ is as given.



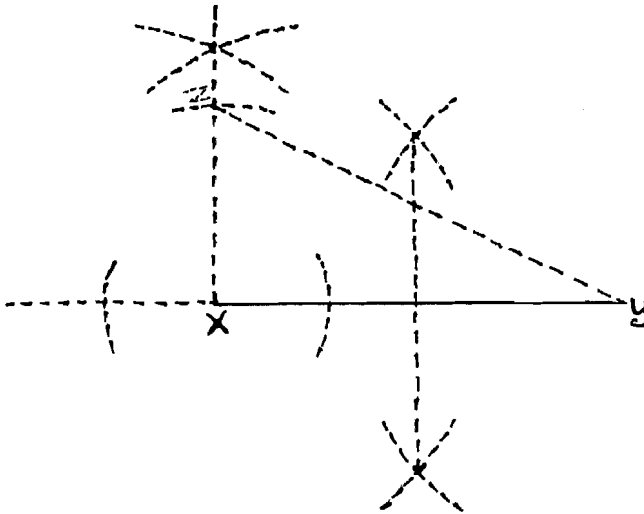
4. At Y on line XZ construct an angle WYZ which will equal 75° .



5. Construct an isosceles triangle ABC in which $\angle B$ is the vertex angle and AC is the base.



6. On line YX construct a right triangle XYZ in which XZ is equal to $\frac{1}{2}$ of XY.



TEST II

Published by
Bureau of Educational Measurements
Kansas State Teachers College, Emporia

Possible score 122

FORM A.

PLANE GEOMETRY

Time: 40 minutes.

By Ida S. Becker, Americus, Kansas

STUDENT'S

SCORE

Name Age Grade

School Teacher

Town State Date

PART I

(Possible score, 30)

Score

DIRECTIONS: In each of the following statements a word, phrase, or number has been omitted where the stars (****) are placed. Write the appropriate word, phrase, or number on the line below the statement.

1. A closed curve all points of which are equidistant from a point within called the center is a ****.
2. A theorem which follows immediately as a consequence of some other theorem is call a (an) ****.
3. The sum of the interior angles of a polygon of n sides is ****.
4. The locus of all points equidistant from the sides of an angle lies on the **** of the angle.
5. A triangle with no two sides equal is a (an) ****.
6. Two angles which have a common vertex and a common side between them are called ****.
7. An angle whose size is between the size of a right angle and that of a straight angle is a (an) ****.
8. Two angles whose sum is equal to a right angle are ****.
9. The perpendicular drawn from the vertex of a triangle to the opposite side is called a (an) ****.
10. The size of the smaller angle formed by the hands of a clock at 5 p. m. is ****.
11. The complement of a 57° angle is ****.
12. In a theorem the line of reasoning leading to a conclusion is called the ****.
13. Alternate interior angles of parallel lines are ****.
14. Triangles whose sides are respectively perpendicular are ****.
15. A straight line which intersects a circle in two points is called a (an) ****.
16. An angle formed by two radii of a circle is called a (an) ****.
17. A polygon whose sides are tangents to a circle is called a (an) ****.
18. A figure formed by an arc and its chord is called a (an) **** of a circle.
19. An equation whose members are equal ratios is called a (an) ****.
20. The first and fourth terms of a proportion are called ***.
21. When the segments of one line have the same ratio as the corresponding segments of another line, the two lines are said to be divided ****.
22. The area of a **** is equal to half the product of the diagonals.
23. An angle formed by the intersection of two chords within a circle equals numerically ****.

24. The **** of a chord passes through the center of the circle.
25. An angle formed by a tangent and a chord drawn from the point of tangency equals numerically ****.
26. A parallelogram inscribed in a circle is a ****.
27. The area of a triangle equals ****.
28. Mutually equiangular triangles are ****.
29. If the angles of one polygon, taken in order, are respectively to those of another, taken in order, the polygons are said to be ****.
30. A straight line which joins any two points on a circle and passes through the center is a (an) ****.

PART II

(Possible score, 30)

Score

DIRECTIONS: Work each of the following problems and write the answer on the line below the problem.

1. Find the area of a triangle if its base and its altitude are 9 and 6 respectively.
2. A side of a rhombus is 5; its diagonals are 6 and 8. Find the altitude.
3. Find the fourth proportional to 8, 6, and 4.
4. Find the third proportional to 2 and 4.
5. State the following proportion by inversion $4/8 = 6/12$.
6. If two corresponding sides of two similar polygons are 15 and 20, what is the ratio of their perimeters.
7. How high is a tree that casts a shadow 50 feet long if an 8-foot vertical post casts a shadow 10 feet long at the same time?
8. The hypotenuse of a right triangle is 75, and one side is 60. Find the altitude upon the hypotenuse.
9. Find the side of a square whose diagonal is 12.
10. Find the area of an equilateral triangle if a side is 12.
11. In an isosceles triangle find the area if a leg is 8 and altitude is 5.
12. The angles of an inscribed triangle are 40° , 60° , 80° . The bisectors of the angles meet the circle at x, y, and z. Find the size of the angles of triangle xyz.
13. If two tangents are drawn from a point P to a circle the intercepted arcs when angle P formed by the tangents is 40° .
14. How many degrees are there in each of the angles of an inscribed triangle if the arcs subtended by the sides are in the ratio of 5:7:12.
15. From an external point a secant and a tangent are drawn to a circle. If the tangent is 9 and the secant is 27, what is the internal segment of the secant?

PART III

(Possible score, 32)

Score

DIRECTIONS: In each of the following problems study the figure and the statements of what is given and what is to be proved. Write the steps of the proof on the lines left for this purpose under "Proof." Next find the reason for each step of your proof in the accompanying "List of Reasons," and write its number in the parenthesis following the step. There should be as many steps in a correct proof as there are lines left for them.

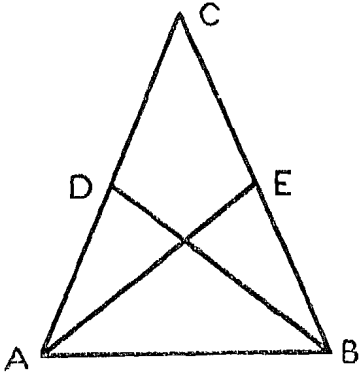


Fig. 1

GIVEN: $AC=BC$

D and E are mid-points of AC and BC respectively.

TO PROVE: $\triangle ACE \cong \triangle BCD$

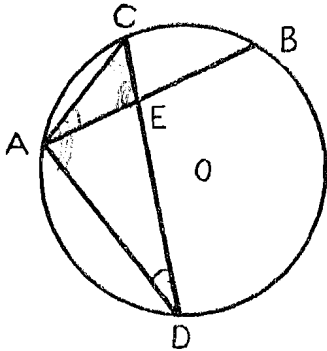


Fig. 2

GIVEN:

In circle O, arc $AC = \text{arc } CB$; AB and CD are intersecting chords.

TO PROVE: $\triangle ACE \sim \triangle ACD$

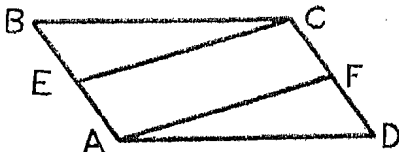


Fig. 3

GIVEN: Parallelogram ABCD;

E is the mid-point of AB; F is the mid-point of CD.

TO PROVE: AECF is a parallelogram.

List of Reasons

1. Hypothesis (given)
2. Identical
3. A quantity may be substituted for its equal in any process.
4. If equals are added to equals, the sums are equal.
5. Halves of equals are equal.
6. Corresponding parts of congruent triangles are equal.
7. Two triangles are congruent if two angles and the included side of one are equal respectively to two angles and the included side of the other.
8. Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.
9. Two triangles are congruent if three sides of one are equal respectively to three sides of the other.
10. The base angles of an isosceles triangle are equal.
11. Two triangles are similar if two angles of one are equal respectively to two angles of the other.
12. Two triangles are similar if their corresponding sides are proportional.
13. The opposite angles of a parallelogram are equal.
14. The opposite sides of a parallelogram are equal.
15. The opposite sides of a parallelogram are parallel.
16. The diagonals of a parallelogram bisect each other.
17. If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.
18. An inscribed angle is measured by one-half its intercepted arc.
19. An angle formed by a tangent and a chord equals one-half its intercepted arc.
20. An angle formed by two chords intersecting within a circle equals one-half the sum of the intercepted arcs.
21. If two parallel lines are cut by a transversal, the exterior-interior (corresponding) angles are equal.
22. If two parallel lines are cut by a transversal, the alternate-interior angles are equal.

Proof	Reasons
1.	1. ()
2.	2. ()
3.	3. ()
4.	4. ()
5.	5. ()
6.	6. ()
7.	7. ()
8.	8. ()
9.	9. ()
10.	10. ()
11.	11. ()
12.	12. ()
13.	13. ()
14.	14. ()
15.	15. ()
16.	16. ()

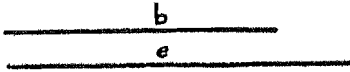
PART IV

Score:

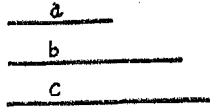
(Possible score, 30)

DIRECTIONS: In these exercises you are to make certain formal constructions. Compass and straight-edge alone are to be used. All necessary construction lines should appear in the completed figure.

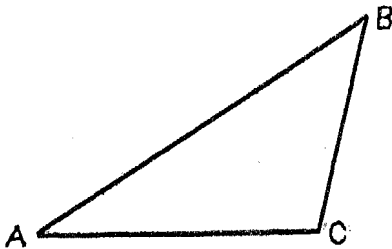
1. Construct an isosceles triangle with a base equal to b and two sides each equal to a .



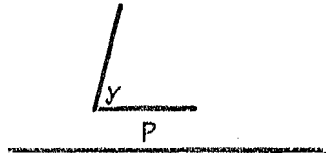
4. Construct a line x which is the fourth proportional to lines a , b , and c .



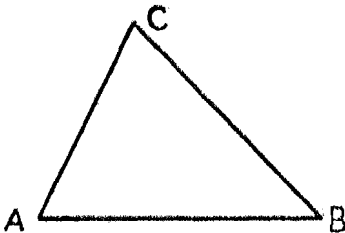
2. Construct the three medians of triangle ABC.



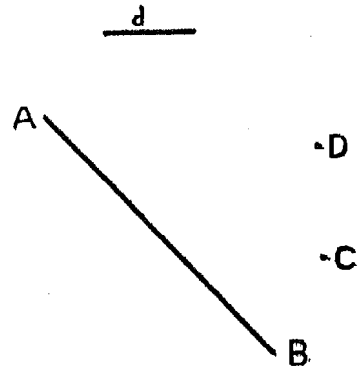
5. Construct a rhombus with its acute angles equal to y and the diagonal joining the vertices of these angles equal to p .



3. Construct a circle inscribed in the triangle ABC.



6. Find all the points at a given distance d from a fixed line AB and equidistant from two given points C and D.



BECKER-SCHRAMMEL
Plane Geometry
KEY
TEST I FORM A

Papers must be scored according to this key. An omission is counted as an error.

Possible score:

Part I	33
Part II	10
Part III	9
Part IV	34
Part V	30
Total score	116

PART I

Allow one point for each correct answer. Possible score, 33.

PAGE 1 PART I Column I	PAGE 1 PART I Column II
1. right angle	13. 90°
2. acute angle	14. equal or supplementary
3. supplementary angles	15. $(n-2)180^\circ$
4. corollary	16. bisector
5. equal	17. surface or plane
6. general or scalene	18. bisect
7. hypotenuse	19. 45°
8. equal	20. vertex
9. transversal	21. equilateral or equiangular
10. median	23. parallel
11. perimeter	24. 60°
12. mutually equiangular	25. $\frac{1}{2}BO$

**PAGE 2
PART I
Column I**

26. 142°

27. 11°

28. 80°

29. 720°

PART II
 Each correct answer counts one point. Possible score, 10.

1. (12)

2. (5)

3. (7 or 17)

4. (7)

5. (9)

6. (20)

7. (1)

8. (14)

9. (15)

10. (21)

**PAGE 2
PART I
Column II**

30. 18

31. 145°

32. 8

33. $a > c$

PART III
 Each correct answer counts one point. Possible score, 9.

1. 33

2. 66

3. 81

4. 38

5. 38

6. 52

7. 40

8. 70

9. 140

Allow one point for each correct answer. Proof and one diagram for each correct answer. For the problems in Figure 5 two diagrams are given. In all problems any answer which is correct, should be credited for the same number of points allowed for a correct answer in this key. Steps 1, 2, and 3 may be stated in any order. Steps 4 and 5 may be stated in any order. Possible score, 34.

1. $\triangle ABC \cong \triangle CBA$ (10)

2. $BD = AE$ (1)

3. $AD = AB$ (2)

4. $\triangle ABC \cong \triangle ABD$ (4)

5. $\triangle ABC \cong \triangle DC$ (1)

6. $\triangle ABC \cong \triangle DC$ (1)

7. $ED = BD$ (2)

8. $\triangle ABC \cong \triangle BDC$ (3)

9. $\angle A = \angle s$ (15)

10. $\triangle ABC \cong \triangle BC$ (1)

11. $\triangle ABC \cong \triangle CA$ (10)

12. $\triangle ABC \cong \triangle DC$ (1)

13. $\triangle ABC \cong \triangle CA$ (10)

14. $\angle A = \angle s$ (18)

15. $\angle A = \angle B = \angle C = \angle D$ (14)

16. $\triangle ABC \cong \triangle BO$ (1)

17. $\triangle ABC \cong \triangle AC$ (10)

18. $\triangle ABC \cong \triangle MB$ (8)

19. $\triangle ABC \cong \triangle MB$ (14)

20. $\triangle ABC \cong \triangle MB$ (8)

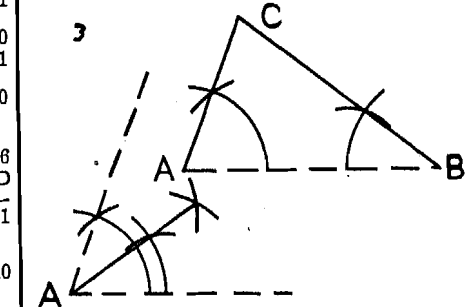
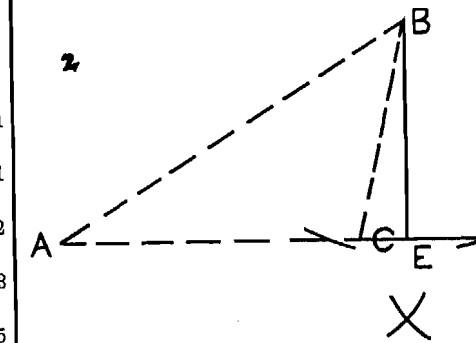
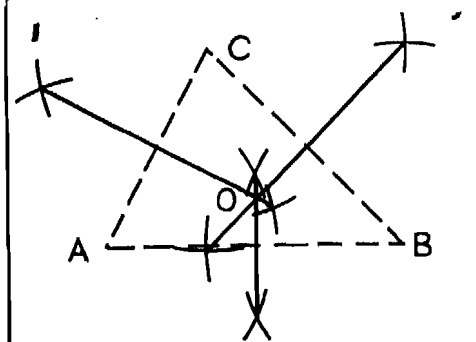
21. $\triangle ABC \cong \triangle MB$ (14)

22. $\triangle ABC \cong \triangle NB$ (11)

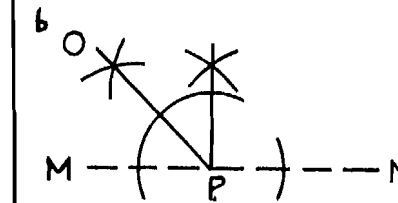
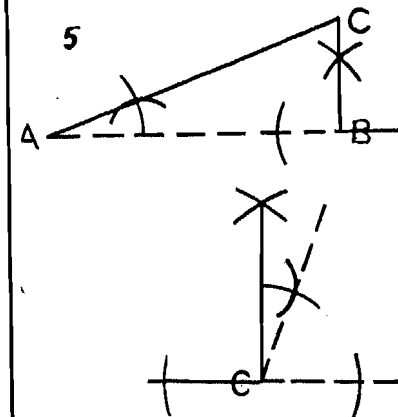
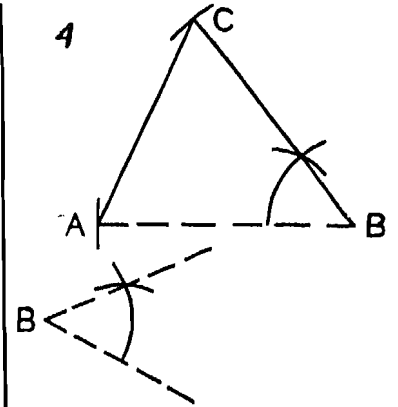
PART V

Allow 5 points for each accurate construction. Score each construction either right or wrong. Allow no partial values. Detailed drawings are given for each. The intent is that each construction shall be made accordingly, by standard construction methods. CARELESS AND INEXACT WORK SHOULD NOT BE CREDITED AT ALL. Where the choice of a radius is arbitrary, the key figures are only suggestive of details. Allow full credit for any other construction that is correct. For No. 1 perpendicular bisectors for any two sides are sufficient. For No. 5 the right angle may be either at A or at B. Possible score, 30.

**PAGE 4
PART V
Column I**



**PAGE 4
PART V
Column II**



KEY
BECKER-SCHRAMMEL
Plane Geometry
TEST II FORM A

Papers must be scored according to this key. An omission is counted as an error.

Possible score:

Part I	33
Part II	33
Part III	32
Part IV	30
Total score	122

PART I

Allow one point for each correct answer. Possible score, 30.

PAGE 1
PART I
Column I

PAGE 1
PART I
Column II

1. circle	13. equal
2. corollary	14. similar
3. $(n-2)180^\circ$, $(n-2)2\text{rt. } \angle$ or $(n-2)\text{st. } \angle$	15. secant
4. bisector	16. central angle
5. scalene triangle	17. circumscribed polygon
6. adjacent angles	18. segment
7. obtuse angle	19. proportion
8. complementary	20. extremes
9. altitude	21. proportionally
10. 150°	22. rhombus
11. 33°	23. $\frac{1}{2}$ the sum of the intercepted arcs
12. proof	

PAGE 2
PART I
Column I

- 24. perpendicular bisector
- 25. $\frac{1}{2}$ its intercepted arc
- 26. rectangle or square
- 27. $\frac{1}{2} BH$, or $\frac{1}{2}$ base \times altitude

PAGE 2
PART I
Column II

- 28. similar
- 29. mutually equiangular
- 30. diameter

PART II

Each correct answer counts two points. Possible score, 30.

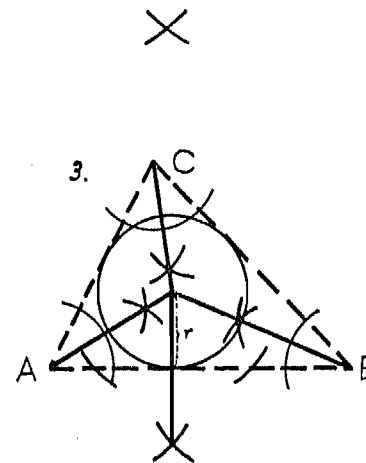
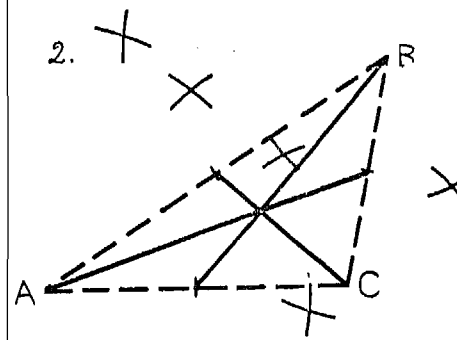
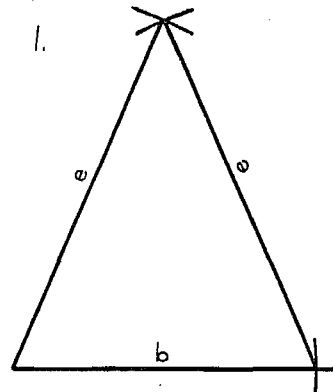
1. 27	9. $6\sqrt{2}$, 8.4, 8.48, or 8.5-
2. 4.8 or $4\frac{1}{2}$	10. $4\sqrt{3}$, 6.92, or 6.9
3. 3	11. 31.2, 31, or $5\sqrt{39}$
4. 8	12. 50° , 60° , 70°
5. $8/4=12/6$	13. 140° and 220°
6. $15/20$, or $\frac{3}{4}$	14. $37\frac{1}{2}^\circ$, $52\frac{1}{2}^\circ$, 90°
7. 46	
8. 36	15. 24

PAGE 3
PART III

Allow one point for each correct step of a proof and one point for each correct reason. Note that for the problem relating to Figure 3 two alternate proofs are given. For each of the problems any other proof, if correct, should be credited for the same number of points allowed for the proof furnished in this key. Steps 1, 2, and 3 may be stated in any order. Steps 5, 6, 7, 8, and 9 may be stated in any order. Possible score, 32.

- 1. $AC = BC$ 1. (1)
 - 2. $CD = CE$ 2. (5)
 - 3. $\angle C = \angle C$ 3. (2)
 - 4. $\triangle ACE \cong \triangle BCD$ 4. (8)
 - 5. arc $AC =$ arc BC 5. (1)
 - 6. $\angle D = \frac{1}{2}$ arc AC 6. (18)
 - 7. $\angle CAB = \frac{1}{2}$ arc BC 7. (18)
 - 8. $\angle D = \angle CAB$ 8. (3)
 - 9. $\angle C = \angle C$ 9. (2)
 - 10. $\triangle ACE \sim \triangle ACD$ 10. (11)
 - 11. $BC = AD$ 11. (14)
 - 12. $AE = \frac{1}{2}AB$ 12. (1)
 - 13. $CF = \frac{1}{2}CD$ 13. (1)
 - 14. $AE = CF$ 14. (5)
 - 15. $AE \parallel CF$ 15. (15)
 - 16. $AECF$ is a parallelogram 16. (17)
- Alternate Proof**
- 11. $BC = AD$ 11. (14)
 - 12. $AB = CD$ 12. (14)
 - 13. $AE = CF$ 13. (5)
 - 14. $AE \parallel CF$ 14. (15)
 - 15. $AECF$ is a parallelogram 15. (17)

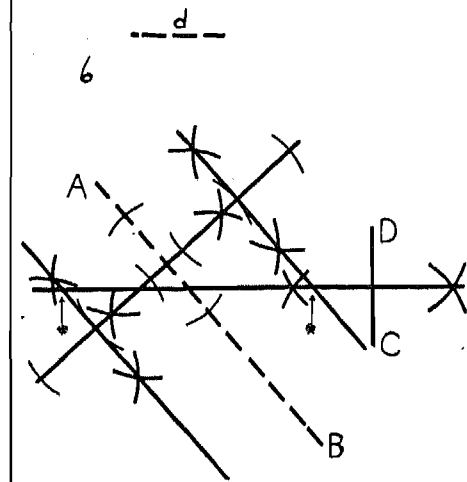
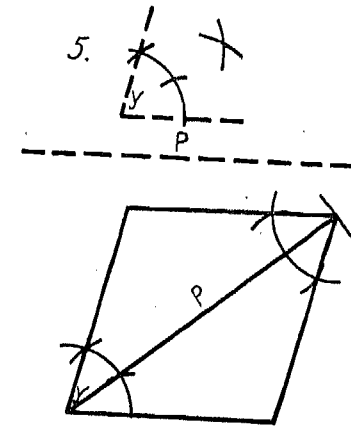
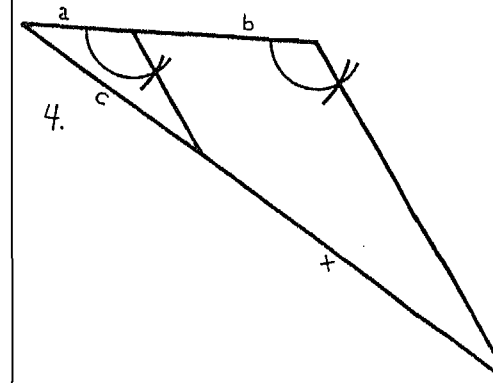
PAGE 4
PART IV
Column I



PART IV

Allow 5 points for each accurate construction. Score each construction either right or wrong. Allow no partial values. Detailed drawings are given for each. The intent is that each construction shall be made accordingly, by standard construction methods. CARELESS AND INEXACT WORK SHOULD NOT BE CREDITED AT ALL. Where the choice of a radius is arbitrary, the key figures are only suggestive of details. Allow full credit for any other construction that is correct. For No. 3 bisectors for any two angles are sufficient. Possible score, 30.

PAGE 4
PART IV
Column II



PLANE GEOMETRY

Test II

Form B

PART I

DIRECTIONS: In each of the following statements a word, phrase, or number has been omitted where the stars (***) are placed. Write the appropriate word, phrase, or number on the line at the left of the statement.

- Concentric circles 1. Circles having the same center but different radii are ***.
- Theorem 2. A statement whose truth may be proved is called a (an) ***.
- 360° or 4 rt. \angle s 3. The sum of the exterior angles of a polygon formed by producing the sides in succession is ***.
- Perpendicular bisector 4. The locus of all points equidistant from the extremities of a line is the ***.
- Isosceles triangle 5. A triangle any two of whose sides are equal is called a (an) ***.
- Right Angle 6. An angle formed by two perpendicular lines is a (an) ***.
- Acute Angle 7. An angle whose size is between the size of a zero angle and that of a right angle is called a (an) ***.
- Supplementary 8. Two angles whose sum is equal to a straight angle are ***.
- Median 9. The line from a vertex of a triangle to the midpoint of opposite side is a (an) ***.
- 120° 10. The size of the smaller angle formed by the hands of a clock at four o'clock is ***.
- 122° 11. The supplement of a 58° angle is ***.
- Interior angle 12. An exterior angle of a triangle is greater than either nonadjacent ***.
- Perpendicular 13. If two consecutive angles of a quadrilateral are right angles, the bisectors of the other two angles are ***.
- Similar 14. Triangles with parallel sides are ***.
- Tangent 15. A straight line which touches a circle in only one place is called a (an) ***.

- Inscribed angle 16. An angle whose vertex is on a circle and whose sides are chords is called a (an) ***.
- Inscribed polygon 17. A polygon whose sides are chords of a circle is called a (an) ***.
- Sector 18. A figure bounded by two radii and their intercepted arc is called a (an) ***.
- Ratio 19. The relation of one number to another is called a (an) ***.
- Means 20. The second and third terms of a proportion are called the ***.
- Proportionally 21. When the segments of one line have the same ratio as the corresponding segments of another line, the two lines are said to be divided ***.
- Square 22. The area of *** is equal to half the square of a diagonal.
- $\frac{1}{2}$ difference of their intercepted arcs 23. An angle formed by two tangents equals numerically ***.
- Perpendicular 24. A line drawn through the center of a circle so that it bisects a chord which is not a diameter is *** to the chord.
- $\frac{1}{2}$ its intercepted arc 25. An inscribed angle equals numerically ***.
- Supplementary 26. The opposite angles of an inscribed quadrilateral are ***.
- Base times altitude 27. The area of a rectangle equals ***
- Similar 28. If an acute angle of one right triangle is equal to an acute angle of another right triangle, the two triangles are ***.
- Similar 29. If two polygons are mutually equiangular and their corresponding sides are proportional, the polygons are ***.
- Chord 30. A straight line joining any two points on a circle is called a (an) ***.

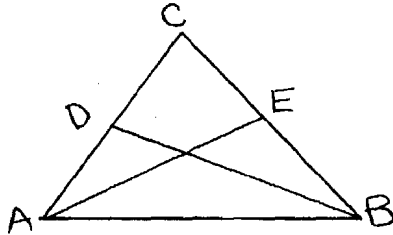
PART II

DIRECTIONS: Work each of the following problems and write the answer on the line at the left of the problem.

- | | |
|----------------------------------|---|
| 16 | 1. Find the area of a triangle if its base and its altitude are 4 and 8 respectively. |
| 7 | 2. A side of a rhombus is 5; its diagonals are 7 and 10. Find an altitude. |
| 6 | 3. Find the fourth proportional to 2, 3, and 4. |
| 8 | 4. Find the third proportional to 2 and 6. |
| $8/5 = 9/x$ | 5. State the following proportion by inversion $5/8 = x/9$. |
| $20/30$ or $2/3$ | 6. If two corresponding sides of two similar polygons are 20 and 30, what is the ratio of their perimeters? |
| 15 ft. | 7. How high is a tree, that casts a shadow 25 feet long, if a 3-foot vertical post casts a shadow 5 feet long at the same time? |
| 12 | 8. The hypotenuse of a right triangle is 25 and one leg is 20. Find the altitude upon the hypotenuse. |
| $5.65, 5.6, 5.7, 4/\sqrt{2}$ | 9. Find the side of a square whose diagonal is 8. |
| $15.57, 15.5, 15.6, 9/\sqrt{3}$ | 10. Find the area of an equilateral triangle if the side is 6. |
| $22.96, 22.9, 23, 4/\sqrt{33}$ | 11. In an isosceles triangle find the area if a leg is 7 and the altitude is 4. |
| $65^\circ - 60^\circ - 55^\circ$ | 12. The angles of an inscribed triangle are 50° , 60° , and 70° . The bisectors of the angles meet the circle at X, Y, and Z. Find the angles of the triangle XYZ. |
| $130^\circ - 230^\circ$ | 13. If two tangents are drawn from a point P to a circle, find the intercepted arcs when angle P formed by the tangents is 50° . |
| $80^\circ - 60^\circ - 40^\circ$ | 14. How many degrees are there in the angles of an inscribed triangle if the arcs subtended by the sides form the ratio 4:6:8. |
| 12 | 15. From an external point a secant and a tangent are drawn to a circle. If the tangent is 8 and the secant is 16, what is the internal segment? |

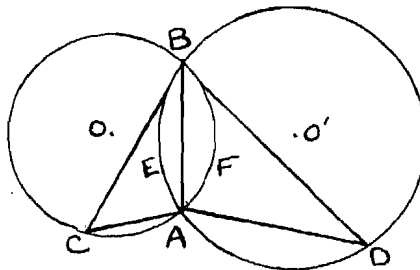
PART III

DIRECTIONS: In each of the following problems study the figure and the statements of what is given and what is to be proved. Write the steps of the proof on the lines left for this purpose under "Proof." Next find the reason for each step of your proof in the "List of Reasons" on the opposite page, and write its number in the parenthesis following the step. There should be as many steps in a correct proof as there are lines left for them.



GIVEN: $\triangle ABC$ is isosceles;
D and E are mid-points
of AC and BC respectively.
TO PROVE: $\triangle ABD \cong \triangle ABE$

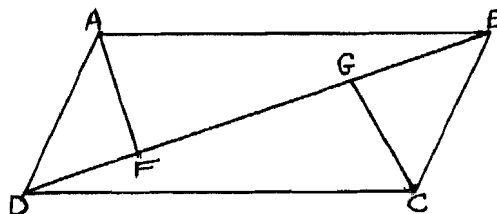
Proof	Reasons
1. <u>$AB = AB$</u>	1. (2)
2. <u>$\angle CAB = \angle CBA$</u>	2. (10)
3. <u>$AD = BE$</u>	3. (5)
4. <u>$\triangle ABD \cong \triangle ABE$</u>	4. (8)



GIVEN: Two intersecting
circles, O and O'; BC and
BD are tangent at B; AB
is a common chord.

TO PROVE: $\triangle ABC \sim \triangle ABD$

5. <u>$\angle CBA$ is meas. by $\frac{1}{2} \widehat{AEB}$</u>	5. (19)
6. <u>$\angle D$ is meas. by $\frac{1}{2} \widehat{AEB}$</u>	6. (18)
7. <u>$\angle CBA = \angle D$</u>	7. (3)
8. <u>$\angle ABD$ is meas. by $\frac{1}{2} \widehat{BFA}$</u>	8. (19)
9. <u>$\angle C$ is meas. by $\frac{1}{2} \widehat{BFA}$</u>	9. (18)
10. <u>$\angle C = \angle ABD$</u>	10. (3)
11. <u>$\triangle ABC \sim \triangle ABD$</u>	11. (11)



GIVEN: ABCD is a parallelogram.
 AF bisects $\angle A$
 GC bisects $\angle C$

TO PROVE: $AF = GC$

12.	<u>$AD = BC$</u>	12.	(14)
13.	<u>$\angle ADF = \angle GCD$</u>	13.	(5)
14.	<u>$\angle ADB = \angle GBC$</u>	14.	(22)
15.	<u>$\triangle ADF \cong \triangle GCB$</u>	15.	(7)
16.	<u>$AF = GC$</u>	16.	(6)

LIST OF REASONS

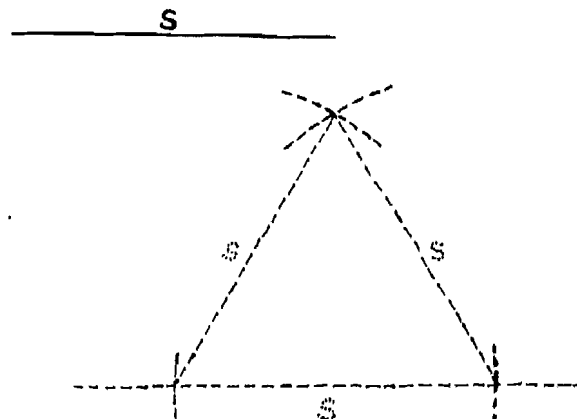
- Hypothesis (given)
- Identical
- A quantity may be substituted for its equal in any process.
- If equals are added to equals, the sums are equal.
- Halves of equals are equal.
- Corresponding parts of congruent triangles are equal.
- Two triangles are congruent if two angles and the included side of one are equal respectively to two angles and the included side of the other.
- Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.
- Two triangles are congruent if three sides of one are equal respectively to three sides of the other.
- The base angles of an isosceles triangle are equal.
- Two triangles are similar if two angles of one are equal respectively to two angles of the other.
- Two triangles are similar if their corresponding sides are proportional.
- The opposite angles of a parallelogram are equal.
- The opposite sides of a parallelogram are equal.

15. The opposite sides of a parallelogram are parallel.
16. The diagonals of a parallelogram bisect each other.
17. If two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.
18. An inscribed angle is measured by one-half its intercepted arc.
19. An angle formed by a tangent and a chord equals one-half its intercepted arc.
20. An angle formed by two chords intersecting within a circle equals one-half the sum of the intercepted arcs.
21. If two parallel lines are cut by a transversal the exterior-interior (corresponding) angles are equal.
22. If two parallel lines are cut by a transversal the alternate-interior angles are equal.

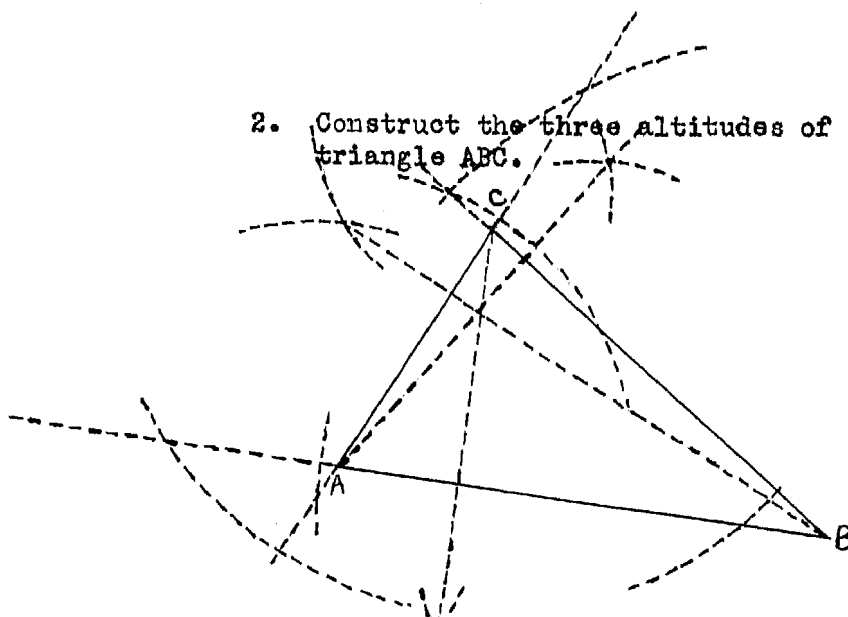
PART IV

DIRECTIONS: In these exercises, you are to make certain formal constructions. Compass and straight edge alone are to be used. All necessary construction lines should appear in the completed figure.

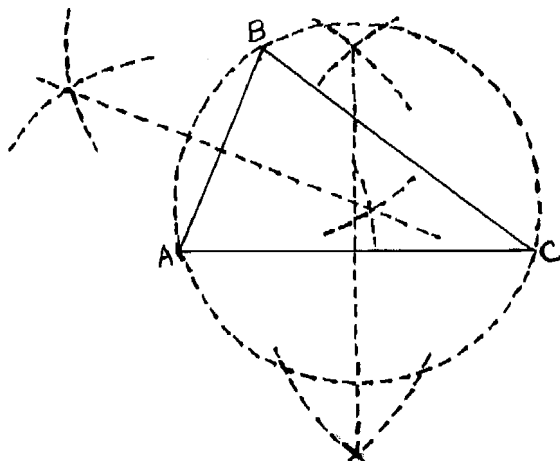
1. Construct an equilateral triangle in which each side is equal to s .



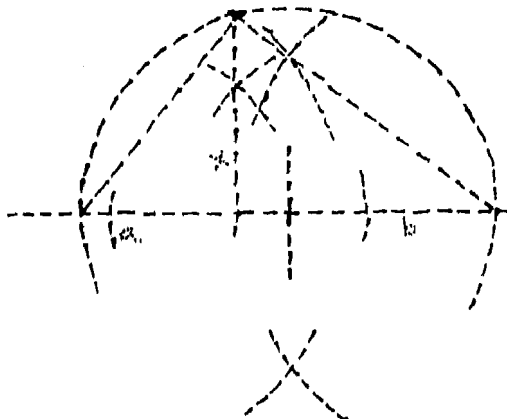
2. Construct the three altitudes of triangle ABC.



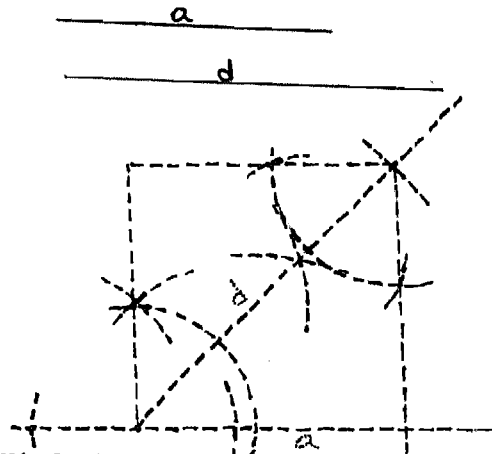
3. Construct a circle circumscribed about the triangle ABC.



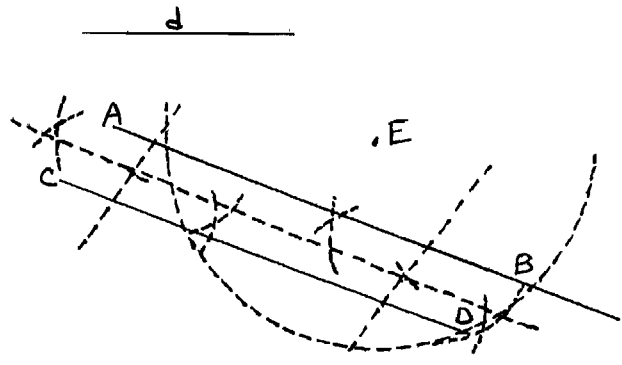
4. Construct a line x which is the mean proportional between lines a and b .



5. Construct a rectangle with the base a and the diagonal d .



6. Find all the points equidistant from two parallel lines AB and CD , and at a given distance d from a given point E .



CHAPTER II

VALIDITY OF THE TEST AND CRITERIA EMPLOYED

One of the most important single facts about a test is the degree of validity which it possesses. Validity has been variously defined as: the degree to which a test measures what it is intended to measure; the general worthwhileness of an examination; and, in general, the degree to which a test parallels the curriculum and good teaching practice.

The criteria utilized in the validation of the tests of this study include the following:

1. An analysis of the content of a number of plane geometry textbooks
2. An analysis of the report of the recommendations of the National Committee on Mathematics Requirements
3. An analysis of the requirements of the College Entrance Examination Board
4. An analysis of the Kansas Course of Study
5. The judgment of competent persons examining and using the texts

Analysis of Textbooks

For the purpose of determining the textbook content, a study of six

leading texts in plane geometry was made. The texts* were the following: Strader and Rhoads, Plane Geometry; Sykes and Comstock, Plane Geometry; Avery, Plane Geometry; Seymour, Plane Geometry; Wells and Hart, Modern Plane Geometry; and Clark-Otis, Modern Plane Geometry.

In the study made of these textbooks it was found that, in general, the books contain the following divisions: introduction, rectilinear figures, circles, areas and proportions, similar polygons and proportional magnitudes, regular polygons, constructions, and loci.

A check was made to discover to what extent each item of the test is used in the six geometry books that were analyzed. It was found that twenty-eight of the thirty-three items in Part I, Form A, were used in the six books; three items were used in five books; and two items, in four books. In Part I, Form B, there were twenty-seven of the thirty-three items in five books, two items in four books, and three items in three books.

Of the thirty items in Part I of Test II, Form A, twenty-six were

* William W. Strader and Lawrence D. Rhoads, Plane Geometry, John C. Winston Co., Chicago. 1927. 399 pages.

Mabel Sykes and Clarence E. Comstock, Plane Geometry, Rand McNally & Co., Chicago. 1923. 322 pages.

Royal A. Avery, Plane Geometry, Allyn and Bacon, Boston. 1925. 319 pages.

F. Eugene Seymour, Plane Geometry, American Book Company, New York. 1925. 333 pages.

Webster Wells and Walter W. Hart, Modern Plane Geometry, D. C. Heath and Company, Boston. 1926. 322 pages.

John R. Clark and Arthur S. Otis, Modern Plane Geometry, World Book Company, Chicago. 1927. 325 pages.

found in all six books; three, in five books; and one, in three books. Twenty-four of the items of Test II, Form B, appear in all six books; four items appear in five books; and two items appear in four books.

The types of problems, proofs, and constructions used in the other parts of the tests were found listed in all of the six books analyzed.

Since the test content conforms so closely to the content of these six reputable textbooks, the conclusion seems warranted that, judged by the textbook-content criterion, the test is a valid instrument for measuring achievement in plane geometry. Moreover, the test should be equally valuable regardless of the textbook or textbooks used by any school.

Courses of Study and Accepted Requirements

The validity of the tests is insured not merely by the experimental usage, textbooks, and detailed statistical analysis to which the various forms were subjected but also by the fact that the tests were based on the Kansas Course of Study. The Kansas Course of Study was used as a basis for selecting the items because the tests were being used chiefly in Kansas and because the writer had taught geometry for eleven years in Kansas.

The test was also checked as to content and terminology against the recommendations of the National Committee on Mathematics and the requirements of the College Entrance Examination Board. Since the content and terminology of the test are the same as those recommended by the National Committee and the College Entrance Examination Board, the conclusion seems justified that the test is adaptable not only to the schools of Kansas but also to the schools of other states. The conclusion is further warranted

by the fact that the Kansas textbook gives special emphasis to the points included in the recommendations of the National Committee on Mathematics and the requirements of the College Entrance Examination Board; and the test was based chiefly on the Kansas text.

It is believed that the test as a whole will commend itself to teachers and students because of its thoroughness.

Judgment of Competent Persons

While the tests were being constructed, they were subjected to the criticisms of competent teachers, supervisors, and specialists in test construction. Many valuable suggestions were utilized. After the preliminary forms of the tests had been administered, additional criticisms were available. These were likewise made use of in making the tests more valid and valuable instruments for measuring student achievement.

Summary

As a basis for establishing the validity of the content, the test of this study was checked against six textbooks of geometry; the report of the Recommendations of the National Committee of Mathematics; the requirements of the College Entrance Examination Board; and the Kansas State Course of Study. It was subjected to the criticisms of teachers and supervisors of mathematics and of test construction specialists. Since the content of the test is in agreement with these reputable criteria, a high degree of validity has been assured.

CHAPTER III

RELIABILITY OF THE TEST

Meaning

The second important quality in a standardized test is its reliability. Reliability refers to:

1. The degree to which a device measures consistently whatever it does measure
2. The degree of accuracy of measurement
3. The amount of confidence that may be placed in the score on a test as a measure of some ability of a pupil

If a test is valid, it is considered to be reliable; but a reliable test does not necessarily possess high validity.

Objectivity

The principal means of guaranteeing reliability in a test is by objectivity of scoring or evaluating and by the character of extent of sampling included in the test items. When these factors are adequately controlled in test construction, the consistency with which the test measures the capacity of those taking it may be statistically determined.

That objectivity is a prime essential for reliability of measurement is clearly demonstrated by the study made by Starch* several years ago. It had previously been supposed that marks could be much more objectively given for geometry than for intangible subjects such as English and history. Starch* secured 115 teachers of high school mathematics to grade

* Daniel C. Starch and Edward C. Elliott, "Reliability of Grading Work in Mathematics," School Review. Vol. XXI, April 1913. pp. 254-59.

the same geometry paper. The marks assigned to this paper ranged from 28% to 95%. Thus it was clearly demonstrated that the ordinary examination is highly subjective, even in as concrete a subject as mathematics. Such examinations possess very little reliability because of the subjective element in scoring.

It is believed that the tests of this study measure the pupil's ability and that the scores are unadulterated by factors which represent the psychological reactions of the teacher. This is true because the items are thoroughly objective and the scoring is done by means of a carefully prepared, objective scoring key. In the second place a test, to be reliable, should contain a wide sampling of the subject matter upon which the pupil is to be tested. The larger the sampling, the more complete will be the analysis of the knowledge of the pupils tested. The longer the test, the greater will be the reliability.

Since the tests consist of completion questions covering definitions and theorems, numerical problems, proof problems, and constructions, the student is given a sampling of practically all types of work covered in geometry. The construction problems are weighted five points each, as it takes longer to make a construction than it does to answer a completion question for which the examinee needs to write but one word, or a number.

The tests of this study were built to be given in a 40-minute class period because most high school classes are from 40 to 60 minutes in length. By having items which may be answered by a minimum of writing, the pupil is able to record his responses to a large number of items in the time allowed.

It is believed that in a test of this type the student perhaps will

do first the part of work he likes best or the part on which he feels himself to be most proficient. Later he will return to the other parts. This is especially noticeable on the papers of the slower students who omitted some of the questions. In this manner each will be able to make a showing in proportion to his knowledge or ability.

For each part of the test instructions which tell the pupils in simple language what they are to do are furnished; examples are given when a new type of test item is employed. Such a method of furnishing directions insures further that all pupils have a fair chance to make a creditable showing according to their ability and tends to provide greater reliability for the instrument of measurement.

Reliability Coefficients

The reliability of a test is usually determined by finding the correlation between scores of the same students on the different forms of the test. The coefficient thus obtained when applied to a test is known as its reliability coefficient.

For determining the reliability of the tests of this study various correlations were made. The coefficients of reliability obtained are shown in Table III. It will be observed that between Forms A and B of Test I a reliability coefficient of $.71 \pm .012$ was obtained. Between Forms A and B of Test II, the coefficient obtained was $.73 \pm .014$. All of the correlations computed for this study were worked by the Pearson Product-Moment Method by the use of the Otis Correlation Chart.

To determine the coefficients of reliability of each test individually, self-correlations were determined for Test I, Parts I and 5

against Parts 2, 3, and 4; and for Test II, Parts 1 and 3 against Parts 2 and 4. The coefficients obtained by these combinations were "stepped up" by the use of the Brown-Spearman formula. The coefficients obtained are also shown in Table III.

TABLE III
RELIABILITY COEFFICIENTS OF THE TESTS

	Reliability coefficients	Number of cases
Test I Form A; Form B	.71 \pm .012	506
Test II Form A; Form B	.73 \pm .014	526
Test I Form A, Parts 1,5;2,3,4	.70 \pm .028	300
Form B, Parts 1,5;2,3,4	.75 \pm .024	300
Test II Form A, Parts 1,3;2,4	.71 \pm .026	300
Form B, Parts 1,3;2,4	.78 \pm .027	300
Average	.74 \pm .034	300

Read table thus: The reliability coefficient between Form A and Form B of Test I is .71 with a probable error of .012. There were 506 cases.

It will be noted that the average of the six coefficients of reliability computed is .74 \pm .034.

The significance of a reliability coefficient is somewhat debatable; however, the following statement may be generally accepted:

A reliability coefficient ranging from

- .95 to .99 is seldom attained in standard tests;
- .90 to .94 is highly desirable in testing;
- .80 to .89 represents some of the best tests now on the market;
- .70 to .79 represents many tests now on the market;
- .69 and below is not satisfactory for individual measurement. Tests with such a reliability coefficient are used in group testing.*

Thus it will be seen that the tests of this study possess sufficient reliability to make them valuable instruments for measuring class and individual attainment.

Quartile Consistency

In addition to the foregoing studies in regard to the reliability of the tests, an analysis was made to determine how many students were consistent in their quartile rankings throughout all four of the tests given. This study was based on 100 papers, selected at random, but including some from each school. It was found that 31% ranked in the same quartile three times and deviated by only one quartile the fourth time; 14% ranked in the same quartile twice and in the quartile immediately above or immediately below twice; 27% ranked in three quartiles but did not deviate by more than two quartiles; and 2% ranked in all four of the quartiles.

Since the majority of students ranked in the same quartile or deviated from perfect consistency by only one quartile, it must be conceded not only that the tests possess a high degree of reliability but

* Henry Lester Smith and Wendell William Wright: Test and Measurements. Silver, Burdett and Company, Chicago, 1928. p. 37.

also that they are valid instruments for predicting future success in the field of plane geometry.

Summary

Because the tests of this study are thoroughly objective and are unadulterated by factors which represent the psychological reactions of the teacher, they measure the pupils ability and attainment in geometry.

Because they give the student a sampling of practically all types of work covered in geometry, they insure further that all pupils have a fair chance to make a creditable showing according to their ability and tend to provide a high reliability of measurement.

The reliability coefficients are sufficiently high to make the tests valuable instruments for measuring class and individual attainment.

Since the majority of students ranked in the same quartile or deviated from perfect consistency by only one quartile, it is believed the tests possess a high degree of reliability.

CHAPTER IV

AN ANALYSIS OF STUDENT RESPONSES ON THE VARIOUS TEST ITEMS

An analysis of the students' responses to the several items of these tests reveals that there was a large diversity from item to item in the degree of accuracy of the responses. This fact means, of course, that the items were not equal in difficulty. An error study of approximately 520 papers was made on both forms of the test, to determine the degree of difficulty of the various items. Of the total items on the tests, none was answered correctly on all the papers.

The number of items that were incorrectly answered by various degrees of frequency is shown in Tables IV, V, VI, and VII. The actual range of incorrect and omitted responses on the items was 94.4%.

On Test I, Form A, only 2 items, numbers 2 and 4, were incorrectly answered or omitted by 4% or less, while questions 12 and 19 were incorrectly answered or omitted by 75% to 79%. On Form B of the same test, one item, number 5, was incorrectly answered or omitted by 4% or less; and question 25 was answered incorrectly or omitted by 90% to 94%. No item was incorrectly answered or omitted by more than 94% on Test I and 95.8% on Test II.

This diversity tends to show that the test is valid because, since none of the questions is answered correctly or incorrectly by all, it is evident that the test differentiates among students at both ends of the distributions.

A check on the various columns of Table IV reveals that on Parts I and II of Test I a larger proportion of items was answered incorrectly by a small percentage of students than was the case on Parts III, IV, and V. In other words, there was a higher degree of efficiency on the parts

TABLE IV

FREQUENCY OF INCORRECT AND OMITTED ITEMS
OF TEST I, FORM A

Percent	Items answered incorrectly and omitted					
	Part I	Part II	Part III	Part IV		Part V
				Statements	Reasons	
0- 4	--	2,4	--	--	--	--
5- 9	1,2,7,18	--	--	--	--	--
10-14	13,26	9	--	--	--	--
15-19	5,21	1,5,10	--	--	--	--
20-24	3,10,20,33	3	--	2	2	--
25-29	23,28	7	6,7	3	3	--
30-34	6,11,14,16	6	1	10	10	--
35-39	22	--	--	--	--	3
40-44	4,9,15,29,30	--	--	4	--	2
45-49	19,24,31	--	9	5,6,12	4,6,12	--
50-54	25	--	4	1,9	5	--
55-59	--	--	2,3,5,8	7	1,7,9	--
60-64	--	--	--	8	8	1,4
65-69	8,27	--	--	--	--	6
70-74	32	8	--	11,13,15	11	--
75-79	12,17	--	--	--	13,15,16	5
80-84	--	--	--	16	17	--
85-89	--	--	--	--	--	--
90-94	--	--	--	--	--	--
95-99	--	--	--	14	14	--

Read table thus: In Part II, two items were missed by less than 5%. These items were numbered 2 and 4.

TABLE V

FREQUENCY OF INCORRECT AND OMITTED ITEMS
OF TEST I, FORM B

Percent	Items answered incorrectly and omitted					
	Part I	Part II	Part III	Part IV		Part V
				Statements	Reasons	
0- 4	--	5	--	2	--	--
5- 9	5,6,9,13,18,20	3,7	--	1	1	--
10-14	--	4	--	4	--	--
15-19	1,2,3,8,26,28	--	--	3	3,5	--
20-24	7	10	--	5	2,11	--
25-29	10,17,19,21	1,2	2	7	7	--
30-34	4,24	8	3,6	11	4	--
35-39	11,15,22,30	6	4	13	13	--
40-44	12,31	--	1	--	--	1
45-49	32,33	--	--	17	--	3
50-54	14	--	--	--	--	2
55-59	23,27	9	8	8,9,10,12	9,12	--
60-64	16	--	7,9	16	8,17	6
65-69	--	--	--	6,14	10,14,16	--
70-74	--	--	5	15	6	4,5
75-79	29	--	--	--	15	--
80-84	--	--	--	--	--	--
85-89	--	--	--	--	--	--
90-94	25	--	--	--	--	--

Read table thus: In Part II, one item was missed by less than 5%. That item was number 5.

TABLE VI
 FREQUENCY OF INCORRECT AND OMITTED ITEMS
 OF TEST II, FORM A

Percent	Items answered incorrectly and omitted				
	Part I	Part II	Part III		Part IV
			Statements	Reasons	
0- 4	13	--	--	--	--
5- 9	1,30	3	--	--	--
10-14	20,22,27	1,5	4	4	1
15-19	4,7,11,21	--	1	1	--
20-24	2,6,9,16,19,28	--	--	--	--
25-29	5,8,15	6,7	3	3	--
30-34	10,25	--	--	--	2
35-39	--	4	--	--	4
40-44	14	--	2	--	--
45-49	18,24,26	--	6,10	2,6	--
50-54	3,12	--	15	--	--
55-59	17	--	5	5	--
60-64	23,29	--	13	10	--
65-69	--	--	9	15	--
70-74	--	2,9	7,8,12,14	7,8,9,13	--
75-79	--	14,15	--	12	3
80-84	--	12,13	--	--	5
85-89	--	8	11	11	6
90-94	--	10,11	--	14	--

Read table thus: In Part I, one item was missed by less than 5%. That item was number 13.

TABLE VII

FREQUENCY OF INCORRECT AND OMITTED ITEMS
OF TEST II, FORM B

Percent	Items answered incorrectly and omitted				Part IV
	Part I	Part II	Part III		
			Statements	Reasons	
0- 4	--	--	--	--	--
5- 9	15,24,25	--	--	--	1
10-14	1,5,7,16,30	1,3,5	4	1	--
15-19	2,8,9,11,20,21,27,29	--	--	--	--
20-24	12,28	--	1	--	3
25-29	3,10,14	6	2,3	2,4	--
30-34	4,17,19	7	--	--	--
35-39	--	--	16	--	2
40-44	22	--	12	3	--
45-49	6	4,15	--	16	--
50-54	--	--	--	12	--
55-59	23	--	15	--	4
60-64	--	--	13	--	--
65-69	13	8,14	11,14	13,15	--
70-74	26	--	--	14	5
75-79	18	2,9,10	--	--	6
80-84	--	--	6,9	11	--
85-89	--	12,13	--	6,9	--
90-94	--	--	5,7,8,10	5,7,8,10	--
95-99	--	11	--	--	--

Read table thus; In Part I, no item was missed by less than 5%. In Part I, three items were missed by between 5% and 9%. The items are 15, 24, and 25.

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standing at the beginning of the test than on those appearing nearer the end of the test.

Two reasons for this suggest themselves. First, the students may have actually been better prepared to answer the items of these parts. If this is true teachers should perhaps place greater stress on content such as is included in the later parts. Secondly, the students may have answered the earlier items more consistently and, on finding themselves limited in time, they selected such items only of the later parts with which they felt themselves thoroughly conversant. This at least would account for the omitted items. Because of the pressure of time they may also have made more errors on the later parts than would otherwise have been the case.

Similar facts may be noted from Tables V, VI, and VII in regard to the other tests. As a rule, a higher degree of efficiency prevailed on the parts appearing near the beginning of the tests than on those appearing later.

Correct and Incorrect Responses

One of the most interesting parts of this study was that of tabulating the incorrect responses used throughout the tests. These data are tabulated in Tables VIII to XI inclusive.

In reading the following tables the item number is given first; this is followed by the correct response; the percent using the correct response; the number of different incorrect responses given; and the incorrect responses given by 5% or more of the students. Below each section of the tables is listed the various additional incorrect responses which were made to each item.

These incorrect responses illustrate one of the main values of the test, that of diagnosing pupils' errors. The teacher can easily utilize the findings in diagnostic and remedial work.

It will be observed from these tables that for most items a large number of different incorrect responses were made. For example for item number 31 of Test I, Form A, fifty-three different incorrect responses were found. For item number 29 of Test I, Form A, the incorrect items numbered forty-eight. It will be noted that in some instances the responses were closely related to the correct answers, but in many cases they reveal clearly that the student was greatly deficient in specific knowledge and in ability to analyze clearly the problem before him.

For example, in the statement, "The bisector of two complementary adjacent angles forms an angle of 45 degrees.," 25% of the students used the response, "90" degrees. This would have been a correct response had the item read "two supplementary angles." Of course the student confused the two words, "complementary" and "supplementary." This confusion can be corrected perhaps by drill on definitions and their applications.

Twenty-six per cent of the group used the word "central angle" in answering the following: "A figure bounded by two radii and their intercepted arc is called a sector." Confusing the words ratio and proportion, 27% used "proportion" when answering "The relation of one number to another is called a ratio." In these cases the confusion between the two terms might be eliminated by a clear explanation of the words and some practice on their use.

In the statement, "The opposite angles of an inscribed quadrilateral are supplementary," 63% used the word "equal." The teacher could help the student to see the absurdity of such a response by having him cut out several different inscribed quadrilaterals and place the opposite angles together. When the student sees his error he is more likely to avoid the wrong answer.

It is unfortunate that it was not possible in this study to consult all the pupils to whom the tests were finally given. Such a procedure would have enlisted their help in discovering the main causes of their difficulty with certain problems. Work of this kind must be left to the classroom teacher, who can and must direct his own remedial work to those examples and problems most often missed, extending the discussion to the less troublesome points as far as time permits.

An analysis of the results from these tests leads to the suggestion that geometry teachers should give greater attention to the fundamentals in the subject and that they should require a greater mastery of these fundamentals. Similar studies to those reported above should be made by each teacher in order to determine the degree of teaching efficiency, as well as student attainment, on each of the several parts.

TABLE VIII
CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
Part I					
1	right angle	92	16		
2	acute angle	93	10		
3	supplementary angles	80	10	right angle	9
4	corollary	61	18	converse	10
5	equal	84	12		
6	scalene	66	19	isosceles	11

* The following are the incorrect responses made by less than 5% of the students. For most of the answers the spelling has been corrected. For some of the responses, however, it was impossible to determine what word the student intended to use, or the word was so badly misspelled that it could not be credited. Such answers are listed below as given by the student and are placed in quotation marks.

Item 1. Right triangle, vertical, adjacent angle, parallelogram, 90° angle, bisector, perpendicular, straight angle, complementary angles, straight line, triangle, secant, radii, square, same plane and equilateral.

Item 2. Complementary angle, obtuse angle, zero angle, reflex angle, right angle, adjacent angles, straight angle, 90° angle, and 45° angle.

Item 3. Complementary, supplements, triangle, obtuse angle, straight angle, adjacent angle, vertical angle, equal adjacent supplementary and equal adjacent angles.

Item 4. Postulate, axiom, proof, fact, proposition, hypothesis, conclusion, theorem, tool, follow-up, second proof, segment, statement, principal, "cosent," "capolliary," and given.

Item 5. Bisected, 45° , complementary angles, 90° , supplementary angles, adjacent angles, scalene, 180° , right, bisectors, and unequal.

Item 6. Obtuse triangle, rhombus, equilateral, acute triangle, irregular triangle, hypotenuse, scales triangle, regular triangle, zero, triangle, trapezoid, right triangle, polygon, quadrilateral, straight angle, secant, unequal, and equiangular.

TABLE VIII (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
7	hypotenuse	96	9		
8	equal	35	19	congruent	36
9	transversal	63		concurrent lines	5
				bisector	5
10	median	80	22	altitude	9
11	perimeter	67	26	side	5
12	mutually equiangular	21	19	equal	11
				similar	5
				equiangular	6
				congruent	42

Item 7. Base, vertice, equal side, legs, supplement, acute, third angle, adjacent, and altitude.

Item 8. Not equal to 180° , coincident, ray, straight angles, adjacent, complementary, similar, exactly alike, intersect, perpendicular, parallel, concurrent, corresponding, supplementary, diagonal, vertical angles, polygons, and non-parallel.

Item 9. Segment, perpendicular, line bisector, diagonal, tangent, line segment, bisect, triangle, secant, parallelogram, parallel, trisector, perpendicular bisector, secant, vertex, chord, median, midpoint, helping line, rectangle, and perpendicular line.

Item 10. Perpendicular bisector of the side, bisector, radii, angle bisector, common vertex, altitude, perpendicular, perpendicular bisector, base angle, "medicator," diagonal, radius, hypotenuse, diameter, dimensions, vertex, and helping line.

Item 11. Diameter, diagonal, base, arcs, segment, circumference, axiom, arm, chord, angle, altitude, transversal, area, boundary, supplements, hypotenuse, exterior angles, perimeter, radius, legs, length, straight line, shape, distance and degrees.

Item 12. Mutually, coincide, regular, identical, proportionally, corresponding, similar, equal numerically, alike, concurrent, regular polygons, equal in the same order, perpendicular, complementary, and mutually equal.

TABLE VIII (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
13	90°	88	14		
14	equal or supplementary	67	24		
15	$(n - 2) 180^\circ$	58	34	$n - 2(180^\circ)$	6
16	bisector	66	16	perpendicular bisector	6
				vertex	11
17	surface or plane	23	37	angle	6
				line	7
				circle	8

Item 13. Acute, 30° , 45° , complementary, supplementary, 180° , equal obtuse angle, adjacent angle, 95° , less than 90° , 2 to 1, 135° and 360° .

Item 14. Exterior-interior, "ogtecen," parallel, parallelogram, right, similar, adjacent, 90° , straight angles, congruent, complementary, concurrent, corresponding, perpendicular, interior, alternate-interior angles, rhombus, isosceles, equiangular, interior angles, equal, squares, opposite angles, and perpendicular bisector.

Item 15. 228° , 180° , 360° , 90° , 41° , 720° , 45° , N , $(n-2) 180/n$, $(n-2) 140$, $n-2/180$, $360/n$, $180n - 360$, $(n - 360) 180$, $(n - 2)$, $n - 180$, $n-2/n (180)$, $2(180 - n)$, $3n$, $180 - n$, $n+180$, n angles, n -gon, $n 180-360$, $n-180/2$, $90/n$, $n/2$, $n - s$ straight angles, $2 + n$, $n - 360$, $90 - n$, $(n-2) 360$, $n/2 (180)$.

Item 16. Line, center, opposite, sides, hypotenuse, straight, altitude, base, bisector, mid-point, perpendicular, "ortho-center," right, and extremities.

Item 17. Revolution, solid, point, ray, figure, square, path, polygon, transversal, parallel line, straight angle, arc, reflex angle, bisector, straight line, area, curve, moving point, geometric movement, length, perimeter, broken line, degree, position, current, axiom, postulate, parallelogram, line segment, base, quadrilateral or rectangle, obtuse angle, radius and side.

TABLE VIII (continued)
CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
13	90°	88	14		
14	equal or supplementary	67	24		
15	$(n - 2) 180^\circ$	58	34	$n - 2(180^\circ)$	6
16	bisector	66	16	perpendicular bisector	6
17	surface or plane	23	37	vertex	11
				angle	6
				line	7
				circle	8

Item 13. Acute, 30, 45, complementary, supplementary, 180° , equal obtuse angle, adjacent angle, 95° , less than 90° , 2 to 1, 135° and 360° .

Item 14. Exterior-interior, "ogtesen," parallel, parallelogram, right, similar, adjacent, 90° , straight angles, congruent, complementary, concurrent, corresponding, perpendicular, interior, alternate-interior angles, rhombus, isosceles, equiangular, interior angles, equal, squares, opposite angles, and perpendicular bisector.

Item 15. 228° , 180° , 360° , 90° , 41° , 720° , 45° , N, $(n-2) 180/n$, $(n-2) 140$, $n-2/180$, $360/n$, $180n - 360$, $(n - 360) 180$, $(n - 2)$, $n - 180$, $n-2/n (180)$, $2(180 - n)$, $3n$, $180 - n$, $n+180$, n angles, n -gon, $n 180-360$, $n-180/2$, $90/n$, $n/2$, $n - s$ straight angles, $2+n$, $n - 360$, $90 - n$, $(n-2) 360$, $n/2 (180)$.

Item 16. Line, center, opposite, sides, hypotenuse, straight, altitude, base, bisector, mid-point, perpendicular, "ortho-center," right, and extremities.

Item 17. Revolution, solid, point, ray, figure, square, path, polygon, transversal, parallel line, straight angle, arc, reflex angle, bisector, straight line, area, curve, moving point, geometric movement, length, perimeter, broken line, degree, position, current, axiom, postulate, parallelogram, line segment, base, quadrilateral or rectangle, obtuse angle, radius and side.

TABLE VIII (continued)
CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
18	bisect	6	13		
19	45°	54	17	22 $\frac{1}{2}$ ° 90°	5 25
20	vertex	78	24		
21	equilateral or equiangular	83	13	isosceles	8
22	trapezoid	63	17	parallelogram	15
23	parallel	73	9	perpendicular	9
24	60°	53	28	equal	12
				45°	6
				30°	11
				acute	17

Item 18. Divide, intercept, cut in half, coincide, segment, divide equally, separate, draw perpendicular, 22 $\frac{1}{2}$ °, divide into congruent parts, inscribe, 180° and bisector.

Item 19. 27 $\frac{1}{2}$ °, 60°, 180°, 12°, 360°, 30°, 135°, 45°, 35°, 120°, 80°, 90° -n, 65°, 36°, and 15°.

Item 20. Apex, third, acute, side, angle bisectors, obtuse, base, opposite, high places, right, vertical angles, non-adjacent interior, median, hypotenuse, straight or hypotenuse, top, vertice, included, 90°, adjacent, common, leg and 30°.

Item 21. Congruent, equal, regular, quadrilateral, concurrent, supplement, equal, equal distant, right, scalene, acute, and equilateral.

Item 22. Polygon, rhombus, quadrilater, rectangle, isosceles, scalene, equiangular, isosceles trapezoid, bisector, isosceles quadrilateral, quadrilateral, pentagon, rhomboid, triangle, square, and congruent.

Item 23. Median, opposite, 1/2, bisector, congruent, not equal and equidistant.

Item 24. 22 $\frac{1}{2}$ °, 40°, right, 54°, straight line, 22°, 90°, 65°, 6°, 20°, 55°, complementary, 66 $\frac{2}{3}$ °, 1/6° larger angle, obtuse angle, 10°, 75°, perpendicular, 35°, 60°, 41°, 80°, 36°, 15°, less than a right angle, 77 $\frac{3}{4}$ ° and 25°.

TABLE VIII (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
25	$\frac{1}{2}BC$	48	28	BC	5
26	142°	87	18	$\frac{1}{4}BC$	5
27	11°	33	36	148	28
28	80°	73	21	180	5
29	720°	59	48	360	11
30	18	58	40	9	5
31	145	52	53		

Item 25. $\frac{1}{8}$, BC, $\frac{1}{3}BC$, AB, 2, $\frac{1}{2}AB$, 8, 75° , $\frac{1}{4}AB$, $\frac{1}{2}BC$, $\frac{1}{8}BC$, respectively, angle, doubles, $\frac{5}{2}BC$, $\frac{1}{4}$, $\frac{1}{4}BY$, 2BC, triangle, AY, $\frac{1}{2}BC$, $\frac{1}{2}XY$, $\frac{1}{2}$, XY, $\frac{1}{16}BC$ and AB/BC .

Item 26. 140° , 76° , 52° , 45° , 178° , 162° , 42° , 152° , 172° , 38° , 142° , 62° , acute, 150° , 19° , 128° , 90° and 132° .

Item 27. 22° , 16° , 158° , 112° , 10° , 79° , 12° , 56° , 90° , 15° , 34° , 7° , 67° , 78° , 132° , obtuse, 80° , 25° , 32° , 118° , 100° , 124° , 122° , 68° , 28° , 61° , right angle, 113° , 6° , 360° , (69° 4° 90°), 74° , 130° , 103° and 96° .

Item 28. 160° , 85° , 90° , 280° , 120° , 40° , 360° , $\frac{1}{4}$, 180° , $\frac{1}{2}$ of right angle, 76° , 240° , 100° , circle, 36° , 130° , 60° , 50° , $\frac{1}{8}$ 10° and 7° .

Item 29. 420° , 120° , 360° , 90° , 24° , 108° , 900° , 147° , 72° , 60° , 45° , 1440° , 10° , 80° , $51\frac{3}{7}^\circ$, $n - 2$ ($\frac{1}{2}n/n$), 360° 2° , 180° , 724° , $n - 2(180)$, 772° , 864° , 240° , $128\frac{4}{7}^\circ$, 540° , $180^\circ - n/2$, 3240° , 36° , 270° , 2160° , 620° , 40° , 480° , 960° , 52° , 440° , 1220° , 320° , $360^\circ(n - 2)$, $(6 - 2)180^\circ$, 135° , 140° , 150° , 160° , 1020° and 780° .

Item 30. 16, 80, 7, 9, 160, 100, 22, 5, 12, 14, 10, 8, octagon, 360, 13, 3, 3240, 1512, 15, 113, 6, 20, n polygon, 79, 158, 38, $(20 - n)36$, 10 sides, hexagon, 19, 160 n, $(n - 2)360$, 70, 3 sides, 32, 4 sides, 6 sides, and pentadecagon.

Item 31. 105, 80, 135, 335, 120, 65, 150, 55, 8, 148, 100, 25, 45, 630, 1050, 125, 205, 160, 305, 301, 35, 155, 15, 325, 5, 102, 85, 108, 95, 350, 115, 38, 1, 795, 225, 330, 90, 120, 245, 505, 165, 125, 285, 11, 75, 110, between 21 and 20, between 5 and 31, 142, 685, 85, 130, and 235.

TABLE VIII (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
32	8	37	57	5	5
33	a > c	79	11	2 = c	5
Part II					
1	parallel	84	10	complementary supplementary	5 7
2	coincide	98	6		
3	equal or right angles	78	13		
4	equal	98	3		
5	interior angle	81	12		
6	supplementary	83	12	parallel complementary	5 7

Item 32. 31, $4\frac{1}{2}$, 18, 7, 15, $2\frac{1}{2}$, 10, 26, 20, 23, $4\frac{1}{3}$, $6\frac{1}{2}$, $15\frac{1}{2}$, 5, 22, 5 to 31, 9, 59, 13, 16, 21, 12, 149, 135, 148, 17, 118, Between 6 and 31, 36, $16\frac{1}{2}$, 6, 31, Between 5 and 31, 50 and 31, 8, and $\frac{1}{2}$ third side.

Item 33. $C = \frac{1}{2} A$, equals, is less than C, $a - B - C$, A, angle, $\frac{1}{2}$, $A = C$, perpendicular, $B = C$, and is not equal.

Part II

Item 1. Complementary, tangent, perpendicular, supplementary angles, right angles, equal, coincide, area, vertical angle and acute angle.

Item 2. Polygon, secant, interior angle, equal and proof.

Item 3. Area, complementary, acute, interior angle, supplementary, parallel, obtuse, vertical angle, perpendicular, chord, tangent and isosceles.

Item 4. Interior angle, right angles, and exterior angle.

Item 5. Equal, right angles, acute angles, supplementary, exterior angles, obtuse, complementary, vertical angle, area, interior angles, trapezoid, and isosceles.

Item 6. Coincide, perpendicular, equal, acute, right angles, verticals angles, perimeter, trapezoid, tangent, isosceles, obtuse, exterior angle.

TABLE VIII (continued)
CORRECT AND INCORRECT RESPONSES TEST I, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
7	acute	73	11	equal	9
8	perpendicular	26	14	complementary	9
				right angles	5
9	polygon	87	7	complementary	9
				equal	30
10	tangent	83	12	chord	5
				secant	8

Item 7. Right angles, exterior angles, obtuse, isosceles, supplementary, interior angle, parallel, acute and perpendicular.

Item 8. Supplementary, acute, area, parallel, obtuse, circle, exterior angle, 180° , polygon, isosceles and vertical angles.

Item 9. Coincide, rhombus, tangent, circle, interior angle, trapezoid isosceles, and area.

Item 10. Secant, chord, perpendicular, coincide, circle, rhombus, right angles, perimeter, radii and trapezoid.

TABLE IX
CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
Part I					
1	adjacent angles	83	13		
2	obtuse angle	85	7	acute angle	14
3	complementary	85	9	supplementary	6
4	theorem	68	15	acute	7
				corollary	6
				axiom	11
5	equal	93	8		
6	isosceles triangle	94	5		
7	legs	80	16	perpendicular	6

* The following are the incorrect responses made by less than 5% of the students. For most of the answers the spelling has been corrected. For some of the responses, however, it was impossible to determine what word the student intended to use, or the word was so badly misspelled that it could not be credited. Such answers are listed below as given by the student and are placed in quotation marks.

Item 1. Vertical, base angles, supplementary, complementary, equal angles, consecutive angles, straight angles, mid-point, right angles, diagonal, equilateral, included and mutually equiangular.

Item 2. Obtuse, supplementary angle, complementary, 90° , straight and reflex angle.

Item 3. Right angle, straight angle, adjacent angles, 45° , 90° , equal and complementary.

Item 4. Postulate, fact, similar, theory, hypothesis, proposition, statement, converse, apothem, principal, positive, given, and conclusion.

Item 5. Equal and supplementary, 90 and 90, 180, complementary, 90, acute, obtuse, adjacent and equal.

Item 6. Equilateral, right triangle, parallelogram, obtuse and scalene.

Item 7. Acute angles, parallel, altitude, bases, equal, hypothesis, common sides, leg and arm, hypotenuse+leg, vertices, complementary, right isosceles triangle, adjacent, sides and arm.

TABLE IX (continued)
CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
8	congruent	83	12	equal	9
9	parallel lines	94	5		
10	altitude	74	12	median	18
11	diagonal	62	18		
12	congruent	58	20	regular equal	7 21
13	180°	93	12		
14	equal or supplementary	50	16	right angles	22
15	360°	64	20	180 $360/n$	5 6

Item 8. Mutually equal, rhombus, similar, concurrent, coinciding, complementary, unequal, intersecting, current, tangent, and mutually equilateral.

Item 9. Identical, perpendicular lines, non-parallel, rays and concurrent.

Item 10. Perpendicular, perpendicular bisector, axiom, bisector, straight, angle bisector, transversal, hypotenuse, diagonal, right triangle and line.

Item 11. Chord, diameter, transversal, median, mutually equiangular, altitude, adjacent, bisector, tangent, perpendicular, vertex, secant, hypotenuse, equal, perpendicular bisector, parallel, perimeter and locus.

Item 12. Parallel, mutually equal, regular, similar, mutually congruent, equal or supplementary, concurrent, equilateral, general, congruent, regular polygons, complementary, equal or similar, mutually regular, mutually related, mutually equiangular and supplementary.

Item 13. Side, 120° , 160° , 360° , 180° , 90° , 190° , extremities, "astit," straight, sum of other two minus 180 and the fourth.

Item 14. Acute, 90° , supplementary, perpendicular, adjacent, verticals, parallel, congruent, complementary, right triangle, congruent, straight angles, 360, concurrent and reflex.

Item 15. 720° , 90° , 76° , 540° , $n(n - 2^\circ)$ (180°), equal, $180/n$, $(n-2)180^\circ$, $n-2/n$ (180°), complementary, $n(n-2)$, $(n-2)$, $n-360^\circ$, $n-2$, supplementary, perpendicular, $(n-2)$, $n/2$ (360°), $n(180^\circ)$.

TABLE IX (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
16	perpendicular bisector	38	32	center bisector	6
17	line	74	14	plane	5
18	bisector	91	12		
19	90°	72	14	45°	14
20	hypotenuse	94	7		
21	isosceles	73	12	equilateral	11
22	quadrilateral	63	16	rectangle	7
				square	11

Item 16. Equal, circle, median, perpendicular, midpoint, angle bisector, length, line, parallel, bisector of line, path, vertex line, tangent, square, base, vertice, diameter, center bisector, place or location, surface, locii, transversal, diagonal, locus, point, altitude, circum-center, incenter, middle, and circle with line as center.

Item 17. Surface, circle ray, angle, reflex angle, arc, geometric-movement, position, shadow, area, revolution, magnitude, 180° , motion and figure.

Item 18. Median, surface, equal, altitude, diagonal, bisector, perpendicular, transversal, bisect, vertex, and acute.

Item 19. 90° , 80° , 60° , 180° , obtuse, 30° , 95° , 135° , 35° , $22\frac{1}{2}^\circ$, 360° , 120° and right angles.

Item 20. Hypothesis, vertex, base, perpendicular, supplements, base angles, and hypotenuse.

Item 21. Equilateral, congruent, equal, concurrent, not equal, supplementary, scalene, right triangle, obtuse, acute, adjacent.

Item 22. Equal, hexagon, equilateral, parallelogram, rhombus, trapezoid, quadrant, triangle, pentagon, regular, decagon, "equilitala" octagon and scalene.

TABLE IX (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
23	medians	45	23	bisector altitudes	6 11
24	120°	68	33		
25	$\frac{1}{4}BC$	9	42	$\frac{1}{3}BC$ BC $\frac{1}{2}BC$	6 8 10
26	43°	84	24		
27	55°	44	38	125 110 120	5 12 5
28	60°	83	15	50	9
29	65-35-80	24	43		

Item 23. Mid-point, angle, center, perpendicular bisector, angle bisectors, bisectors of side, sides, parallel, transversal, vertices, legs, locus, diagonals, incenter, orthocenter, perpendicular, trapezoid, extremities, base, line quadrilateral.

Item 24. Obtuse, acute, 60° , straight angle, 20° , 120° , 40° , 135° , 45° , 24° , 440° , 105° , 110° , 30° , 112° , 200° , 130° , 5° , 90° , 150° , 65° , 75° , 95° , 115° , 126° , $22\frac{1}{2}^\circ$, 160° , $112\frac{1}{2}^\circ$; supplementary, 120° , 80° , 70° and 96° .

Item 25. $2\frac{1}{2}$, AB, $\frac{1}{2}AB$, 4AB, BC/18, $\frac{1}{8}BC$, $\frac{1}{6}XY$, XY, $\frac{1}{9}BC$, 2 AB, $\frac{1}{3}AB$, 4 BC, $\frac{5}{6}BC$, $\frac{1}{6}BC$, 2BC, $\frac{1}{6}AB$, $\frac{2}{3}$, $\frac{2}{3}AB$, $\frac{1}{3}$, $\frac{1}{9}AB$, 9 BC, $\frac{2}{3}BC$, $\frac{1}{18}$, $1\frac{1}{6}Y$, $\frac{1}{6}$, $1\frac{1}{3}$, $\frac{1}{3}\frac{3}{32}$, 2, 1.8, $\frac{1}{36}BC$, $\frac{1}{12}BC$, 1, $\frac{1}{12}$, $1\frac{1}{2}BC$, $\frac{1}{9}$, $\frac{1}{36}$, 4, and $1\frac{1}{2}$.

Item 26. 1330° , 53° , 130° , 34° , 43° , 40° , 133° , 33° , $23\frac{1}{2}^\circ$, 47° , 90° , 13° , 83° , 94° , 113° , 122° , 50° , 123° , 57° , 42° , 4° , 22° , 110° and acute.

Item 27. Triangle, 10° , 60° , 160° , 122° , 35° , 40° , 250° , 215° , 70° y, 120° , 180° , 140° , 111° , 150° , 1110° , 50° , 52° , 14° , x 70° , 25° , 22° , 290° , 70° , 20° , 116° , 100° , 210° , 135° , $21\frac{1}{2}^\circ$, $27\frac{1}{2}^\circ$, 145° , 115° , 30° , 103° , and $33\frac{1}{3}^\circ$.

Item 28. 50° , 20° , 90° , 30° , 100° , 300° , 180° , numerically 2 times arc, 9° , 35° , 40° , 160° , 30° , and two points.

Item 29. 60, 30-65-85, 50, 5, 68-33-80, 100-50-30, 70-30-80, 30-60-90, 70, 35, 180, 90-65-35, 65-35-180, 80, 230, 65-35-75, $33\frac{1}{3}$, 130-20, 170-6, 30-65-80, 60-35-50, equal, supplementary, acute, obtuse, unequal 60-45-35, 90-65-25, 3 : 5 : 10, 310, 35-80-63, 314, 80-150, 70-130, 90-30, 95-35-20, 35-65, 80-30, 100-130, 130, 150, 95, 30, 55 and 50-100-130.

TABLE IX (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
30	540	64	40	720 360	5 7
31	40	58	34		
32	11	54	28	8	9
33	160°	53	48		
Part II					
1	right angles	75	13	supplementary	8
2	proof	74	11	given	9
3	equal	94	9	conclusion	9

Item 30. 180, $122\frac{1}{2}$, 900, (n-2) 80, 108, 530, 54, 450, n - 2/6, 550, 520, 72, 60, $51\frac{3}{7}$, 1800, 420, (n-2) 360/n, n/180, 180/5, 1080, 3600, 540, 600, 1440, 40, 580, 780, 324, 3240, 560, 36, 360 - n, 146, 1280, 740, 460, 630, and $5(5-2)$ 180.

Item 31. Equal, 9, 20, 40, 4, n, 8, $17\frac{3}{11}$, 5, 420, 10, 16, 42, 6, 12, 9, 3150, 7, $2\frac{1}{2}$, 38, 3, 80, 44, 30, 15, 11, 1260, 60, 140, 100, 22, 25, 147, 90.

Item 32. $10\frac{1}{2}$, 22, 0, 4, 11, 15, 23, 18, 68, 32, 42, $7\frac{1}{2}$, 12, 13, 7, $11\frac{1}{2}$, 5, $3\frac{1}{2}$, 14, 10, 20, $12\frac{1}{2}$, 16, 1, 6, 172, 8 and 22, between 8 and 21, and 22 or 8.

Item 33. 20° , 33° , 86° , 144° , 8x, 100° , $11\frac{1}{2}^\circ$, 80° , 320° , 32° , 138° , 1° , 30° , 158° , 64° , 36° , 40° , 116° , 172° , $42\frac{1}{2}^\circ$, $22\frac{1}{2}^\circ$, 360° , x 8, 1572° , 178° , 84° , 162° , 140° , 8x less, 45° , 155° , 8n, 94° , 60° , 2880° , $(8-2)$ 180° , $157\frac{1}{2}^\circ$, 22° , $17\frac{7}{9}$, 180° , 1440° , $167\frac{1}{2}^\circ$, 315° , $57\frac{1}{2}^\circ$, 170° , 8-n times 180 and supplementary.

Part II

Item 1. Exterior angle, equal, complementary, perpendicular, four, unequal, acute, interior angle, parallel, five, area and right angles.

Item 2. Proof, exterior angle, complementary, perimeter, perpendicular, tangent, secant, isosceles and interior angles.

Item 3. Supplementary, acute, unequal, complementary, parallel, interior angle, perpendicular, area, and secant.

TABLE IX (continued)

CORRECT AND INCORRECT RESPONSES TEST I, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
4	parallel	88	11	supplementary	6
5	four	98	3		
6	area	64	18	perimeter	16
7	equal	93	5	unequal	6
8	secant	68	7	chord	28
9	obtuse	44	16	unequal	6
				equal	7
				right angles	7
				complementary	5
				acute	5
				supplementary	9
10	isosceles	76	10		

Item 4. Area, conclusion, equal, right angles, complementary, perpendicular, unequal, five, parallel, and four.

Item 5. Five, conclusion and given.

Item 6. Interior angle, exterior, trapezoid, equal, four, tangent, secant, conclusion, unequal, right angles, supplementary, perpendicular, secant, complementary, chord, acute, and given.

Item 7. Four, chord, secant, and five.

Item 8. Tangent, conclusion, unequal, obtuse, complementary, and secant.

Item 9. Four, area, secant, chord, given, perpendicular, isosceles, five, 90° , and perimeter.

Item 10. Parallel, unequal, trapezoid, equal, supplementary, acute, given, right angles, interior angle and five.

TABLE X
CORRECT AND INCORRECT RESPONSES TEST II, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
Part I					
1	circle	95	12		
2	corollary	77	15		
3	(n-2) 180	49	36	180	5
				360	9
				n-2(180)	5
4	bisector	84	16	perpendicular bisector	5
5	scalene	74	16	isosceles	6

* The following are the incorrect responses made by less than 5% of the students. For most of the answers the spelling has been corrected. For some of the responses, however, it was impossible to determine what word the student intended to use, or the word was so badly misspelled that it could not be credited. Such answers are listed below as given by the student and are placed in quotation marks.

Item 1. Sector, center, mid-point, circumscribe, locus, point, radius, regular polygon, concentric, another curve, arc, and polygon.

Item 2. Postulate, proof, axiom, fact, statement, converse, antecedent, apothem, "ad," hypotenuse, conclusion, tool, result, proposition, and 2 step-proof.

Item 3. 120, 60, 90, $180/n$, $180n$, $n-2/n$, $n+2/n(360)$, n , $n-180$, $360/n$, $n-3/180$, $n-2(360)$, $n(n-180)$, $n-2/n(180)$, $180-n/2$, $n-180/2$, $(n-2)180$, $360-n$, $180-n$, $n-2$, $720n$, $360n$, $n/2(360)$, $136/n$, $n-2/180$, n , 180 , $n/2(180)$, $n/2(180)$, $n-180$, $2(n-16)$, vertex, $n-2/720$ and $n(n-2)$.

Item 4. Center, sides, perpendicular, mid-point, vertex, isosceles, size, side, hypotenuse, acute, vertical, base, line vertices, circumference and 360° .

Item 5. Polygon, quadrilateral, obtuse, acute, trapezoid, equilateral, right triangle, plane triangle, rectangle, irregular, sequences, non-equilateral, equiangular, scale, and supplementary.

TABLE X (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
6	adjacent angles	77	17		
7	obtuse angle	86	10	acute angle	7
8	complementary	75	17	acute angle	9
9	altitude	79	10	supplementary	5
10	150°	68	42	median	16
11	33°	82	19	obtuse	5
12	proof	49	22	123°	6
				hypothesis	14
				given	6
				to prove	5

Item 6. Supplementary, central angle, isosceles, vertical angle, complementary, equilateral, inscribed angles, similar, triangles, congruent, vertices, right angles, right triangles, parallel, base angle, bisectors of an angle, and consecutive angles.

Item 7. Complementary angle, supplementary, zero angle, right angle, adjacent angle, 90° , triangle, 135° , and reflex angle.

Item 8. Right angle, 180° , 30° - 60° , 30° , equal, concurrent, straight, adjacent, obtuse, congruent, 90° , equal adjacent angles, complementary adjacent, acute-obtuse, and 45° .

Item 9. Bisector, perpendicular bisector, locus, complementary, right angle, diameter, radius, base and hypotenuse.

Item 10. $67\frac{1}{2}^\circ$, 135° , 350° , 105° , altitude, 360° , 60° , 140° , 25° , 30° , 120° , 70° , 160° , 50° , 125° , 180° , 45° , acute, 170° , 80° , 75° , 100° , supplementary, 185° , 179° , 90° , right angle, 150° , 15° , 88° , 210° , 36° , reflex, 115° , vertex, 72° , 5° , 95° , 80° , 3 o'clock and 140° .

Item 11. 43° , 143° , 301° , 23° , 55° , 30° , 150° , 132° , 3° , 57° , 32° , 147° , 35° , 114° , 28° , $16\frac{1}{2}^\circ$, 42° , and 157° .

Item 12. To find clause, facts, concurrency, proposition, postulate, consequence, steps, conclusion, axiom, corollary, therefore, proof, hypotenuse, why, 33° , complement, reasons, assumptions, proportional.

TABLE X (continued)
CORRECT AND INCORRECT RESPONSES TEST II, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
13	equal	96	3		
14	similar	60	16	congruent	5
				equal	9
15	secant	76	8	right triangle	12
16	central angle	78	17	chord	20
17	circumscribed			inscribed angle	7
	polygon	46	27		
18	segment	54	23	polygon	18
19	proportion	76	17	sector	6

Item 13. Supplementary, equal and parallel.

Item 14. Supplementary, right angles, rectangle, proportional, equilateral, chord, parallelogram, square, complementary, isosceles, bisected, parallel and $5/7 = 1/x$.

Item 15. Radii, tangent, segment, diameter, central angle, complementary, and isosceles.

Item 16. Interior angle, circumscribe, sector, right angle, intercepted, straight angle, inscribed polygon, diameter, secant, central circle, vertical, segment, radius, tangent, acute angle, vertex angle.

Item 17. Inscribed circle, "excribed polygon," square, regular polygon, rectangle, secant, triangle, chord, quadrilateral, tangent, semi-circle, central angle, rhombus, tangent polygon, "subscribe polygon," intercepted angle, circle inscribe by a polygon, equilateral, constructive, exterior angle, scalene, n-gon, hexagon, n-polygon, and trapezoid.

Item 18. Segment, diameter, inscribe angle, secant, perpendicular bisector, minor arc of circle, part, intercepted arc, radii, tangent, quadrant, central angle, proportion, arc, semi-circle, sequence, angle, section, inscribe angle of circle, triangle, chord and "bow."

Item 19. Term ratio, equal proportions, equilateral, segment, proportional, mean proportional, extremes, relationship, unity, section, similitude, equality, identity, fraction, vertex, equally and equal ratio.

TABLE X (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
20	extremes	88	11		
21	proportionally	83	16	equally	9
22	rhombus	86	10		
23	$\frac{1}{2}$ the sum of the intercepted arcs	41	16	$\frac{1}{2}$ its intercepted arc	35
				$\frac{1}{2}$ difference of intercepted arc	10
24	perpendicular bisector	53	14	perpendicular diameter	5
				bisector	6
					14

Item 20. Means, mean proportional, antecedents, alternate, external, consequents, similar, proportion, extremities, fourth proportional and "conconcuse."

Item 21. Equal, proportionally equal, proportion, congruent, respectively, similar, externally, internally, mutually, one-half, square, corresponding, rationally, equal parts, and extremes.

Item 22. Rectangle, parallelogram, trapezoid, one half its arc, triangle, quadrilateral, circle, polygon, right triangle, and circle.

Item 23. Sum of intercepted arc, numerical its intercepted arc, one half, one half larger arc minus arc smaller, difference of intercepted arc, one half product of arc, one half arc plus vertical angle, one sum of vertical angles, similar, one half difference of chords, equal, parallel lines, one half sum, and one half central angle.

Item 24. Center, tangent, mid-point, segments, part, radii, longest, length, bisected and plane.

TABLE X (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM A

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
25	$\frac{1}{2}$ its intercepted arc	69	18	$\frac{1}{2}$ difference of intercepted arc	18
26	rectangle of square	56	17	rhombus	9
27	$\frac{1}{2}$ BH	87	12	equal	5
28	similar	78	8	equilateral	5
29	mutually equiangular	40	14	similar	46
30	diameter	92	8	congruent	5

Item 25. One half its intercepted arc, sum of intercepted arc, one half sum of its arcs, "s t = t e," one half, intercepted arc, 90° , its angle, difference of two arcs, one half sum of exterior, two, segment and tangent, equal, similar, another, one half angles, and one half sum of difference of arcs.

Item 26. Inscribe parallelogram, inscribe polygon, quadrilateral, parallelogram, polygon, inscribed quadrilateral, equilateral, isosceles trapezoid, hexagon, figure, regular polygon, trapezoid, isosceles, equiangular, inscribed rectangle and circumscribed polygon.

Item 27. Equal, $\frac{1}{2} l w$, 180, $B \times h$, $B \times h \times 2$, 360, " s^2/S^2 " " $A^2 = B^2$ ", $l \times w$, sides, $S/T = T/E$, and $\frac{1}{2}abh$.

Item 28. Mutually equilateral, congruent, supplementary, proportional, regular, and equally.

Item 29. Mutually equal, equal, proportional, equiangular, regular, "condurent," sides proportion, not similar, concentric, tangent, perpendicular, proportionally equiangular.

Item 30. Circle, chord, diagonal, secant, radii, perpendicular bisector, perpendicular and altitude.

TABLE XI

CORRECT AND INCORRECT RESPONSES TEST II, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
Part I					
1	concentric circles	86	14	similar	5
2	theorem	82	14	axiom	5
3	360°	74	22	180°	5
4	perpendicular bisector	69	22	bisector	8
5	Isosceles triangle	90	10		
6	right angle	96	11		

* The following are the incorrect responses made by less than 5% of the students. For most of the answers the spelling has been corrected. For some of the responses, however, it was impossible to determine what word the student intended to use, or the word was so badly misspelled that it could not be credited. Such answers are listed below as given by the student and are placed in quotation marks.

Item 1. Equal, incurred, unequal, congruent, inscribed, concurrent, central, "centerceddle," circumscribed, centric, tangent, different, and semi-circles.

Item 2. Corollary, hypothesis, principal, axiom, proposition, problem, congruent, postulate, "quiorem," straight angle, proof, statement, fact, and tool.

Item 3. Supplementary, sum of interior angles, sum of non-adjacent interior angles, path, exterior angle, similar, 720° , 540° , 90° , 60° , 860° , 260° , $360^\circ + 80^\circ$, $360/n$, $180/n$, $n - 18/2$, $n/2(180)$, $(n-2)360$, $(n-180)360$, $30-6-90$, $5-2(180)$.

Item 4. Circle with extreme line as center, circle, parallel of it, center, bisactor of line, radius, diameter, locus, vertex, altitude, perpendicular, two parallels, mid-point, angle bisector, parallellines on each side, proportional, distance from it, two lines, congruent triangles, mean proportional, median, and shortest distance.

Item 5. Triangle, similar triangle, right triangle, corresponding triangle, quadrilateral, equilateral triangle, "equulator," scalene triangle, congruent triangles, right angle.

Item 6. Vertical right angle, triangle, right triangle, straight, isosceles, vertical, square, acute, adjacent angle, 180 and bisector.

TABLE XI (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
7	acute angle	88	8	obtuse	5
8	supplementary	80	17	complementary	5
9	median	85	11	right angles	9
10	120°	71	30	altitude	11
11	122	85	24	obtuse angle	7
12	interior angle	76	11	32	5
13	perpendicular	35	18	angle	16
14	similar	75	17	diagonals	8
				equal	30
				equal	9

Item 7. Supplementary angle, complementary angle, reflex angle, obtuse angle, right triangle, 90° , 45° and right angle.

Item 8. Adjacent angles, 80° 100° , two right angles, acute, equal, obtuse, one revolution, supplementary, right triangle, 90° , 180° , similar, 30° , and acute and obtuse.

Item 9. Bisected angle, angle bisector, perpendicular bisector, 90° , perpendicular, diameter, vertex, parallelogram, 122° , diagonal and equal distant.

Item 10. 20° , 150° , $112\frac{1}{2}^\circ$; 108° , 280° , 140° , 70° , 60° , 20° , 40° , obtuse angle, complementary angle, right angle, acute angle, supplementary angle, 45° , 80° , 90° , 110° , 100° , 240° , 160° , 105° , 12° , 130° , 30° , 135° , 115° , and 180° .

Item 11. 125, 120, 302, 148, 132, 116, 121, 162, 33, 124, 45, 102, 22, 30, 92, 152, 35, 185, 122, 32, 42, 112 and 95.

Item 12. Yes, sides, angles, acute angles, adjacent angles, true, exterior-interior angles, false, interior acute angle, and exterior angle.

Item 13. Different size, acute, right angles, 45, diameters, parallel, straight, concurrent, hypotenuse, complementary parallel, similar, supplementary, altitudes, bisected, proportional and the same.

Item 14. Supplementary, parallelogram, congruent, concurrent, right triangle, parallel, proportional, square, equilateral triangles, right angles, trapezoids, isosceles, perpendicular, corresponding, similar and rectangle.

TABLE XI (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
15	tangent	91	7		
16	inscribed angle	86	22		
17	inscribed polygon	60	19		
18	sector	21	29	segment	15
19	ratio	67	9	central angle	26
20	means	85	7	proportion	27
21	proportionally	81	11	equally	10
22	square	60	10	rhombus	30

Item 15. Radii, chord, vertex, sector, secant, diameter and transversal.

Item 16. Inscribed triangle, interior angle, right, median, isosceles, equilateral triangle, right triangle, arc, secant, 90, central angle, sector, segment, obtuse angle, triangle, inscribed angle, circumscribed, tangent, vertical, polygon, semicircle, = parallelogram.

Item 17. Regular, square, pentagon, circumscribed polygon, quadrilateral, regular polygon, secant, rectangle, parallelogram, hexagon, segment, inscribed quadrilateral, triangle, figure, inscribed angle, rhombus, apothem, heptagon, and equilateral.

Item 18. Triangle, arc, chord, circle, quadrant, diameter, cone, secant, tangent, semi-circle, isosceles triangle, right triangle, central arc, angle, circumscribed, vertical angle, equilateral triangle, minor, rectangle, inscribed angle, quadrilateral, arc angle, interior angle hexagon, apothem orthex, "sesic," and intercepted arc.

Item 19. Equation, comparison, faction, unity, radius, antecedent, equivalent and mean proportion.

Item 20. Mean proportional, antecedent, extremes, corollary, consequence, medians, and proportion.

Item 21. Internally, true, in proportion, numerically, similar, perpendicular, harmonically, mutually, externally and rhombus.

Item 22. Parallelogram, trapezoid, quadrilateral, rectangle, side, triangle, polygon, right triangle, and equal.

TABLE XI (continued)

CORRECT AND INCORRECT RESPONSES TEST II, FORM B

Item	Correct response	% correct response	Number of different incorrect responses*	Incorrect responses made by 5% or more of the students	
				response	%
23	$\frac{1}{2}$ difference of their intercepted arc	42	17	$\frac{1}{2}$ seem of intercepted arc	10
				$\frac{1}{2}$ its intercepted arc	31
24	perpendicular	93	8		
25	$\frac{1}{2}$ its intercepted arc	92	8		
26	supplementary	28	8	equal	63
27	B h or l w	83	17	$\frac{1}{2}$ BH	8
28	similar	76	7	congruent	14
29	similar	84	10		
30	chord	86	8	secant	8

Item 23. Intercepted arc, 90° , one-fourth of a circle, right, one-half sum of angles, difference of intercepted arc, one-half sum of sides, difference between two arcs, one-half difference of tangents, one-half larger arc minus smaller arc, tangent, one-half its chord, equal, similar, one-half different, 180° , and two times intercepted arcs.

Item 24. Radius, tangent, parallel, equal, mean proportional, proportional, secant and bisector.

Item 25. 90° , twice, one-half its chord, equal, 180° , similar and one-half sum of intercepted arcs.

Item 26. Complementary and supplementary, 360° , parallel, right angles, exterior-interior angles, complementary, similar, and bisected.

Item 27. One-half square of its diagonal, 360° , $ab/a'b'$, s^2 , 180° , square of diagonal, diagonal plus diagonal over two, $D^2/4$, 2 sides squared, $a - 2$, $(s/2)^2/\sqrt{3}$, $1 : 2$, parallel, $\frac{1}{2} h (B+B)$, $\frac{1}{2}$ its diagonal and $(n-2) 180^\circ$.

Item 28. Equal, mutually equiangular, regular, acute, segment, complement, equiangular.

Item 29. Mutually equilateral, sides proportionally, equal, regular, congruent, secant, smaller, similitude, in proportion, and in same ratio.

Item 30. Segment, diameter, tangent, sector, identical, arc, and equal.

CHAPTER V

INTERPRETATION AND USE OF TEST RESULTS

The real value of a test is realized only when the results obtained from its use are interpreted adequately. The intelligent interpretation of the results is the factor of chief interest to the teacher.

In an earlier chapter it was pointed out that if maximum reliability is to be attained on a test it is essential that the test be long enough to yield a fairly wide distribution of scores. In other words it is desirable that the scores arrange themselves practically as a normal distribution.

The extent to which the scores on this series of tests form a normal distribution may be seen by noting Figures 1, 2, 3, and 4. From these Figures it will be observed that the range of scores on each test was very large; that there were few extremely high or low scores; and that the modal scores were approximately at the middle of the distribution. When graphically plotted the curve formed, from each distribution of scores, conformed very closely to the theoretical normal curve.

Measures of Central Tendency and Variability

In order to be able to interpret more concretely the scores of a group, the measure of central tendency and variability should be known. These have been computed for each of the series of tests of this study and are listed in Table XII. By means of these measures it is possible to compare the degree of equivalence of the tests and the variability of the achievement of the students.

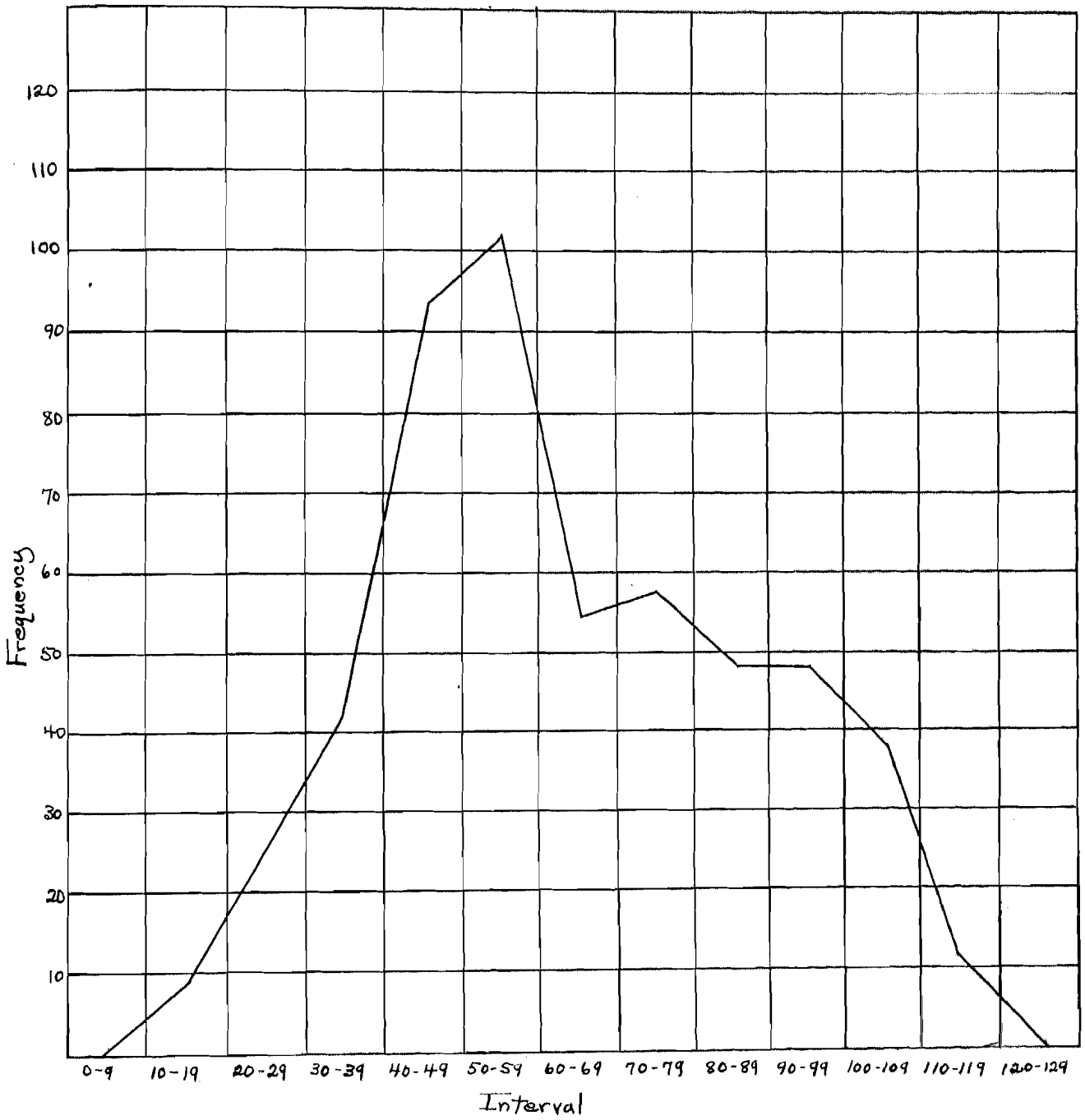


Figure 1. Distribution of Scores on Form A of Test I

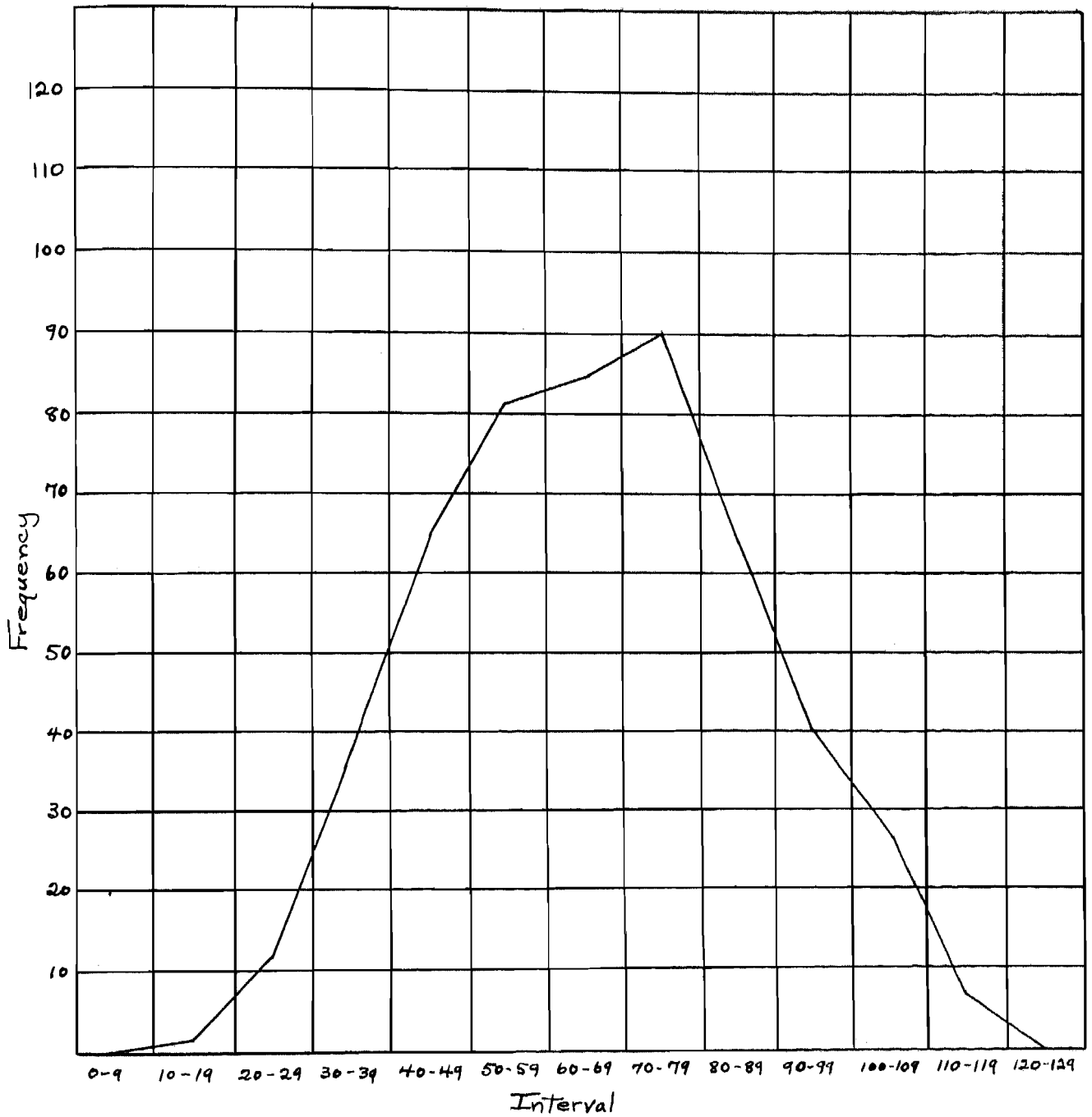


Figure 2. Distribution of Scores on Form B of Test I

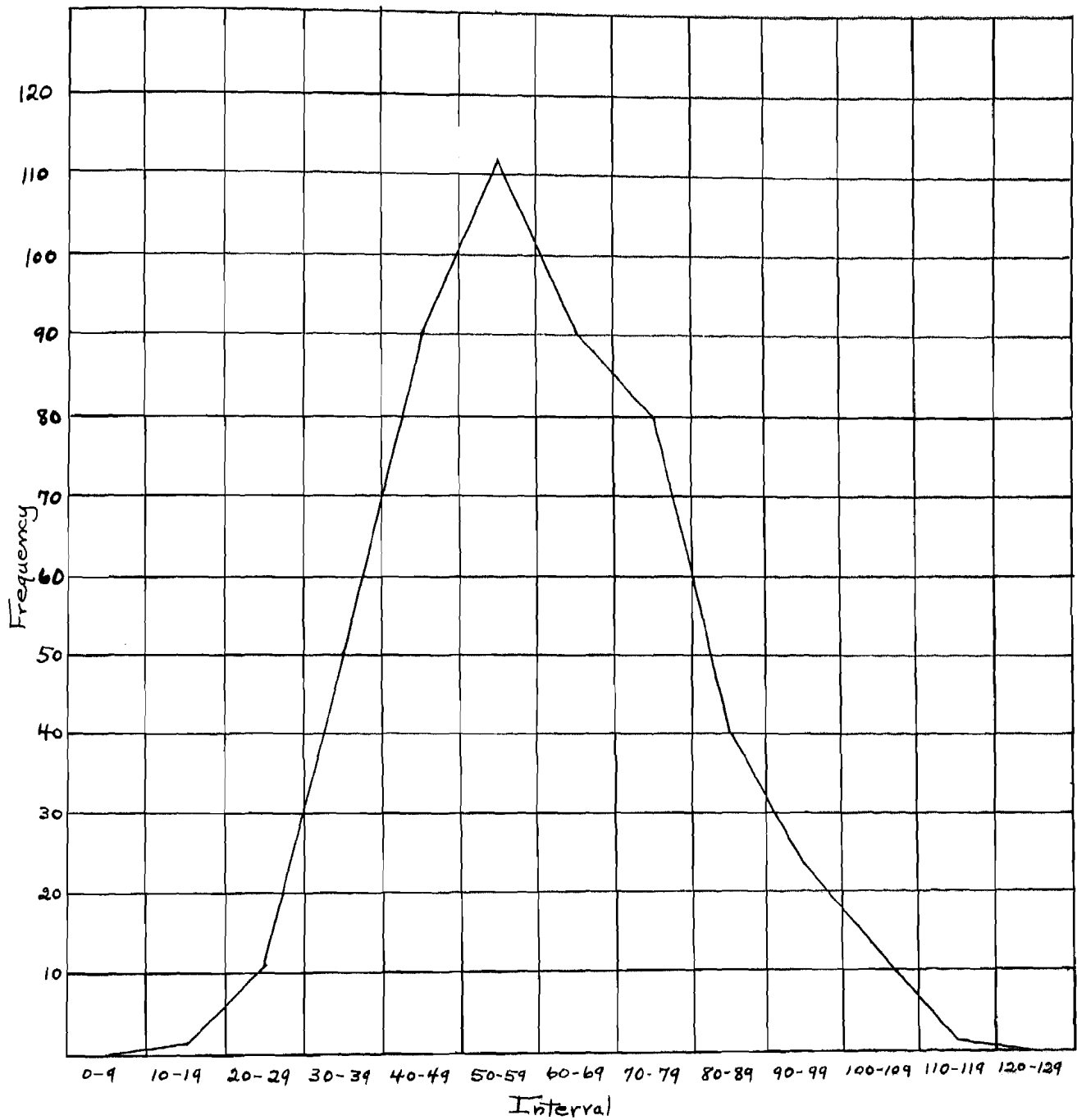


Figure 3. Distribution of Scores on Form A of Test II

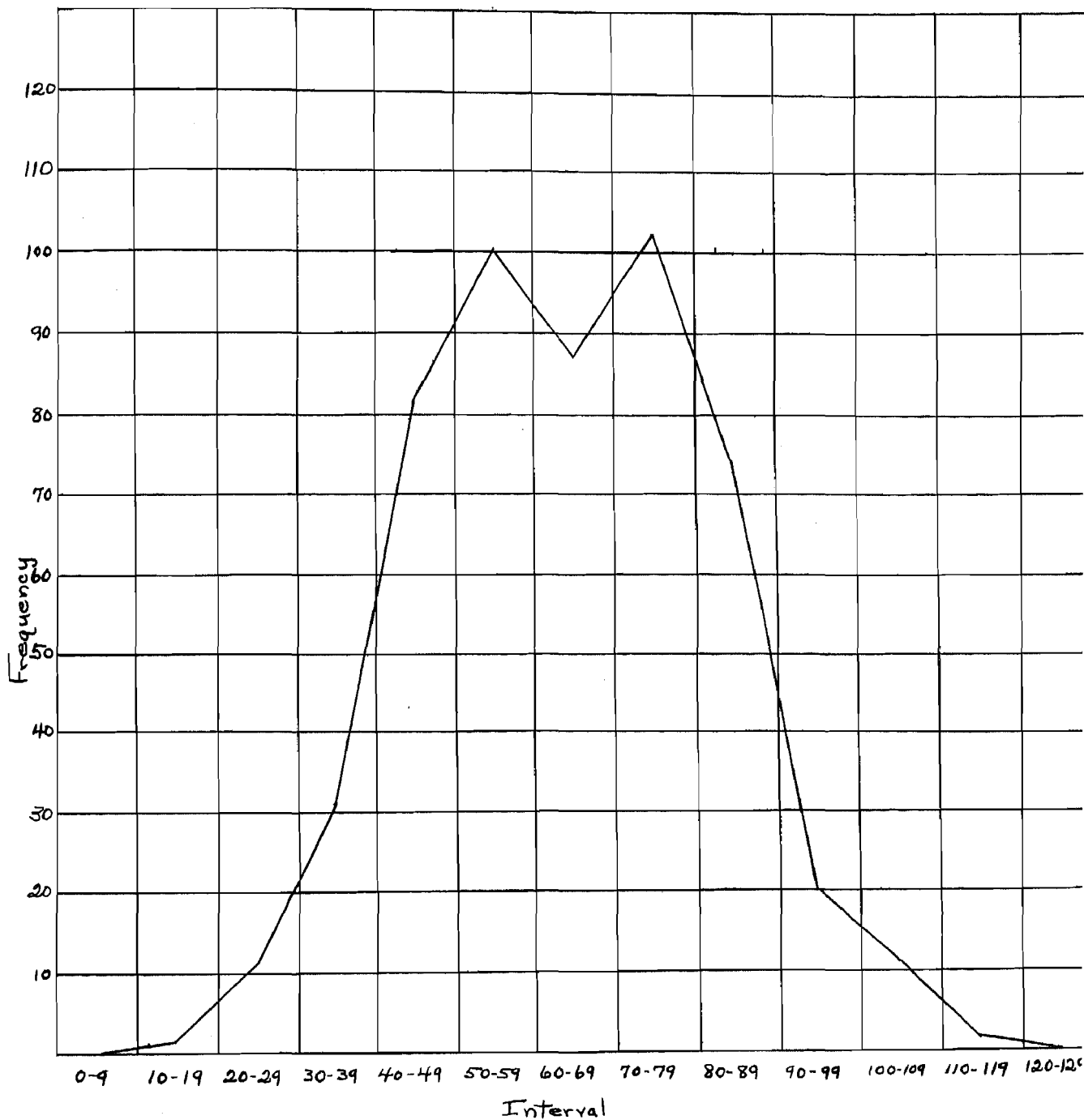


Figure 4: Distribution of Scores on Form B of Test II

No attempt was made in this study to compute any of the measures of central tendency or variability by schools. These measures were computed for the group as a whole. It will be noticed that the number of cases used on Test I and Test II were practically the same.

TABLE XII
MEASURES OF CENTRAL TENDENCY AND VARIABILITY

	Test I		Test II	
	Form A	Form B	Form A	Form B
Number taking test	520	508	522	524
Average	64	67	62	64
Median	58	68	58	63
Mode	52.5	77.5	57.5	72.5
Range	102	98	102	101
Q	18	15	13	11
S.D.	23.7	20.3	18.7	18.1
σ_{ave}	1.04	.90	.82	.79
PE _(mdn)	.99	.83	.68	.62

Read table thus: 520 students took Test I, Form A; 508 students took Test I, Form B; 522 students took Test II, Form A; and 524 students took Test II, Form B.

The fact that the average score in each test did not vary more than 3 points shows a high reliability. In these tests we find a difference of 10 points in the median on Test I and 8 points difference on Test II. Likewise the mode is considerably higher form Form B than for Form A. The cause for these differences will be pointed out later in connection with percentile norms.

The range and the other measures of variability are much the same for the equivalent forms. The measures of reliability of the median and mean are small, showing high reliability of the obtained measures.

Percentile Norms

Percentile norms are of greater value than merely median or mean norms for purposes of comparing the standing of different individuals on the same or different tests and of the same individual at different testing dates.

For instance in these tests the percentile ranks of three students on Test I, Forms A and B, and Test II, Forms A and B, respectively, were as follows:

Bethel	50	55	90	90
Kenneth	70	95	85	90
Harold	45	85	80	55

By the use of such data the teacher is able to check the improvement of the student and also the efficiency of his instruction. In other words, it is much more significant, for example, to know that a student ranks ninetieth percentile on a test than to know merely that he ranks above the group median.

In this study, percentile scores for Form A of Test I and Test II were computed by the Bureau of Educational Measurements of the Kansas State Teachers College, Emporia, for use in connection with the Every Pupil Scholarship Tests. These percentile scores are based on 4823 pupil scores for Test I and 4325 pupil scores for Test II.

Table XIII gives the percentile scores computed by the Bureau of Measurements on Form A of Tests I and II and the scores computed by the writer on both Forms A and B for the schools where both forms were given.

It may be seen from Table XIII that the percentiles of the smaller group were a little higher than those for the entire group. This shows, of course, that the schools comprising the small group were more efficient

TABLE XIII
PERCENTILE SCORES FOR EACH TEST

Percentiles	Test I			Test II		
	Form A		Form B	Form A		Form B
	(1)*	(2)**	(3)**	(1)*	(2)**	(3)**
99 %ile	107	112	112	101	108	104
95 %ile	95	106	103	86	97	93
90 %ile	85	99	96	78	87	86
85 %ile	79	95	90	73	82	83
80 %ile	73	86	86	68	77	83
75 %ile	69	83	82	64	73	77
70 %ile	65	78	79	61	78	75
65 %ile	62	73	76	58	71	72
60 %ile	58	68	73	55	65	70
55 %ile	55	64	70	52	62	67
50 %ile	52	59	67	50	60	69
45 %ile	50	57	64	47	57	66
40 %ile	47	54	61	44	55	63
35 %ile	44	52	58	42	53	60
30 %ile	41	49	55	39	50	53
25 %ile	38	47	52	36	48	50
20 %ile	35	44	42	33	45	47
15 %ile	32	41	44	30	42	44
10 %ile	27	36	40	26	38	41
5 %ile	21	28	36	21	34	35
1 %ile	12	12	25	12	25	26
Number of cases	4823	520	508	4325	522	524

Read table thus: In Test I, Form A, 99% did not exceed 107 or 112; Form B, 99% did not exceed 112. On Test II, Form A, 99% did not exceed 101 or 103; Form B, 99% did not exceed 104.

*Quoted from Bureau of Educational Measurements, Kansas State Teachers College of Emporia; Report of the nineteenth and twentieth Nation-wide Every Pupil Scholarship Test.

** Computed by the writer.

than the large group for which the scores were reported in the Nation-Wide Testing Program. Percentiles on Form B are higher than those on Form A, no doubt because Form B was given within ten days after giving of Form A. The difference presumably is due to practice effect on test mechanics or to intervening study. It is also possible that a discussion of Form A had taken place before the giving of Form B and that the increment is due to added efficiency because of the learning effect.

Median Scores on Each Division of the Tests

In Table XIV are listed the median scores made by the students on each part of each form of Tests I and II. It will be observed that there is almost perfect consistency in respect to these medians between the corresponding parts of the equivalent forms. This indicates that the corresponding parts of the tests are really equivalent in difficulty and in themselves reliable measures of pupil attainment.

TABLE XIV
MEDIAN SCORES FOR EACH PART OF EACH TEST

	Test I			Test II	
	A	B		A	B
Part I	22	23	Part I	22	23
Part II	8	8	Part II	12	12
Part III	5	5	Part III	12	12
Part IV	18	19	Part IV	15	20
Part V	10	10			
Total scores	520	508		522	524

Read table thus: The median score on Test I, Form A, of Part I, is 22; on Test I, Form B, 23; Test II, Form A, 22; and on Test II, Form B, 23.

Part Norms

Norms on the various parts of a test make possible more detailed analyses on the part of the teacher and more exact diagnostic work. From the following citation of the scores of three students on the different parts of the tests, it is seen how one may check back to see on which part of the test the student ranks above or below the median.

			Parts				
			1	2	3*	4*	5
Harold	Test I,	Form A	27	9	2	5	15
		Form B	23	9	6	33	20
	Test II,	Form A	22	18		19	15
		Form B	23	16		14	20
Paul	Test I,	Form A	22	0	3	0	15
		Form B	28	10	8	15	15
	Test II,	Form A	28	14		7	0
		Form B	27	7		12	15

Harold's score on Part 5, Test I, Form A, is 5 points above the norm, and on Form B it is 10 points above the norm. This shows that he ranks above the average in construction work. On Test II his score on both forms is exactly at the norm. Hence it may be concluded that he is consistently efficient in this work.

Paul's scores on Part 4, Test I, Form A is 18 points below the norm, and on Form B is 1 point below the norm. This shows that he ranks below the average on proof work. On Test II, his score on Form A is 5 points below the norm and on Form B it is just at the norm. Hence it may be concluded that he is inefficient in this work.

* In Test II, types of items used in Parts 2 and 3 were combined. Therefore Parts 4 and 5 correspond to Parts 3 and 4 of Test II.

A graphical representation of the scores of the group on Part I of Test I, Forms A and B is given in Figure 5; and for Test II, Forms A and B, in Figure 6. One of the most striking features of these graphs is the lack of symmetry in the figure. There is a piling up of the scores at the high end of the scale or, in other words, the curve is negatively skewed. This indicates that on this part there was a greater proportion of capable students than of very poor students.

It is significant, however, that there is marked conformity between the two graphs on each figure. This shows that the two forms are highly equivalent in respect to accuracy of measurement.

Summary

It must be kept in mind that the mere use of the tests--just giving and scoring them--is not sufficient. The real value of the tests is realized only when the test results are interpreted adequately and teaching practices and procedures are based on them.

The norms should be used for interpreting average achievement of a group. The statistical findings of this study tend to show that the several tests of the series are valid and reliable measures. The percentile scores are most valuable for the interpretation of both individual and class scores.

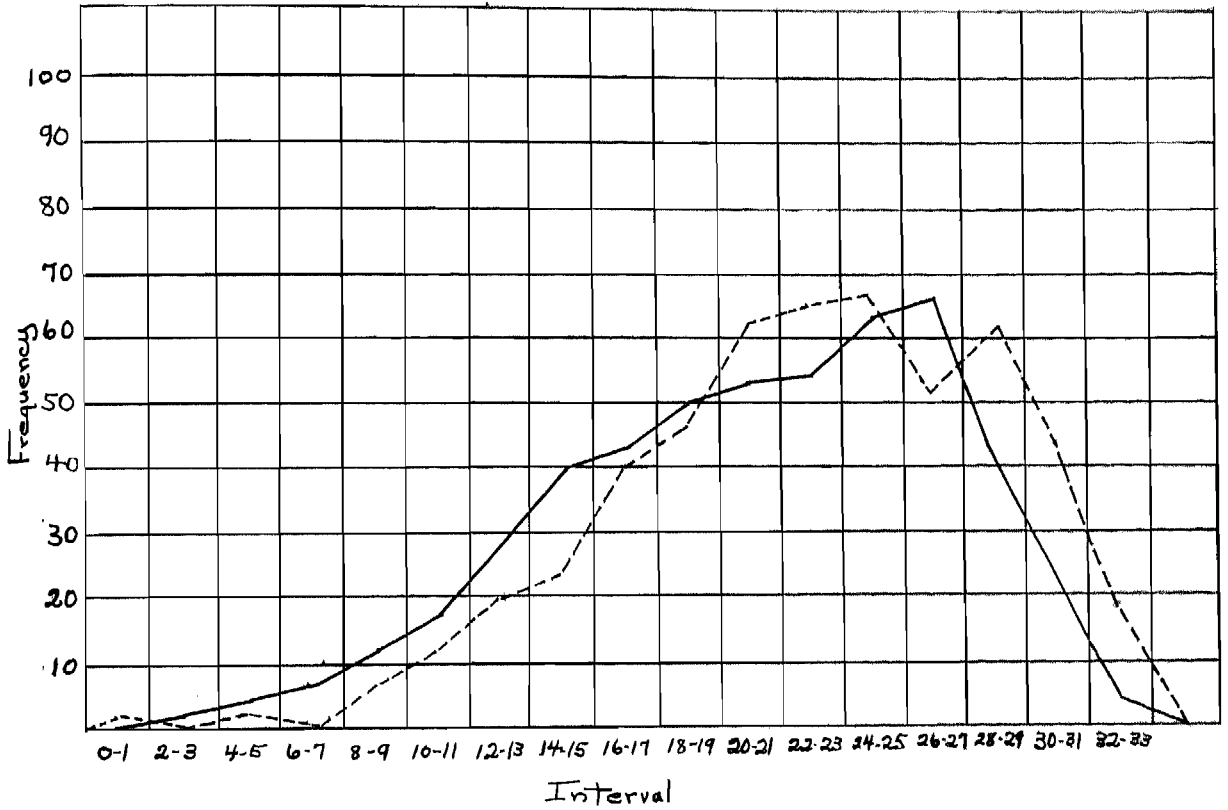


Figure 5. Distribution of Scores on Part I, Forms A and B of Test I

Legend: — Form A - - - - - Form B

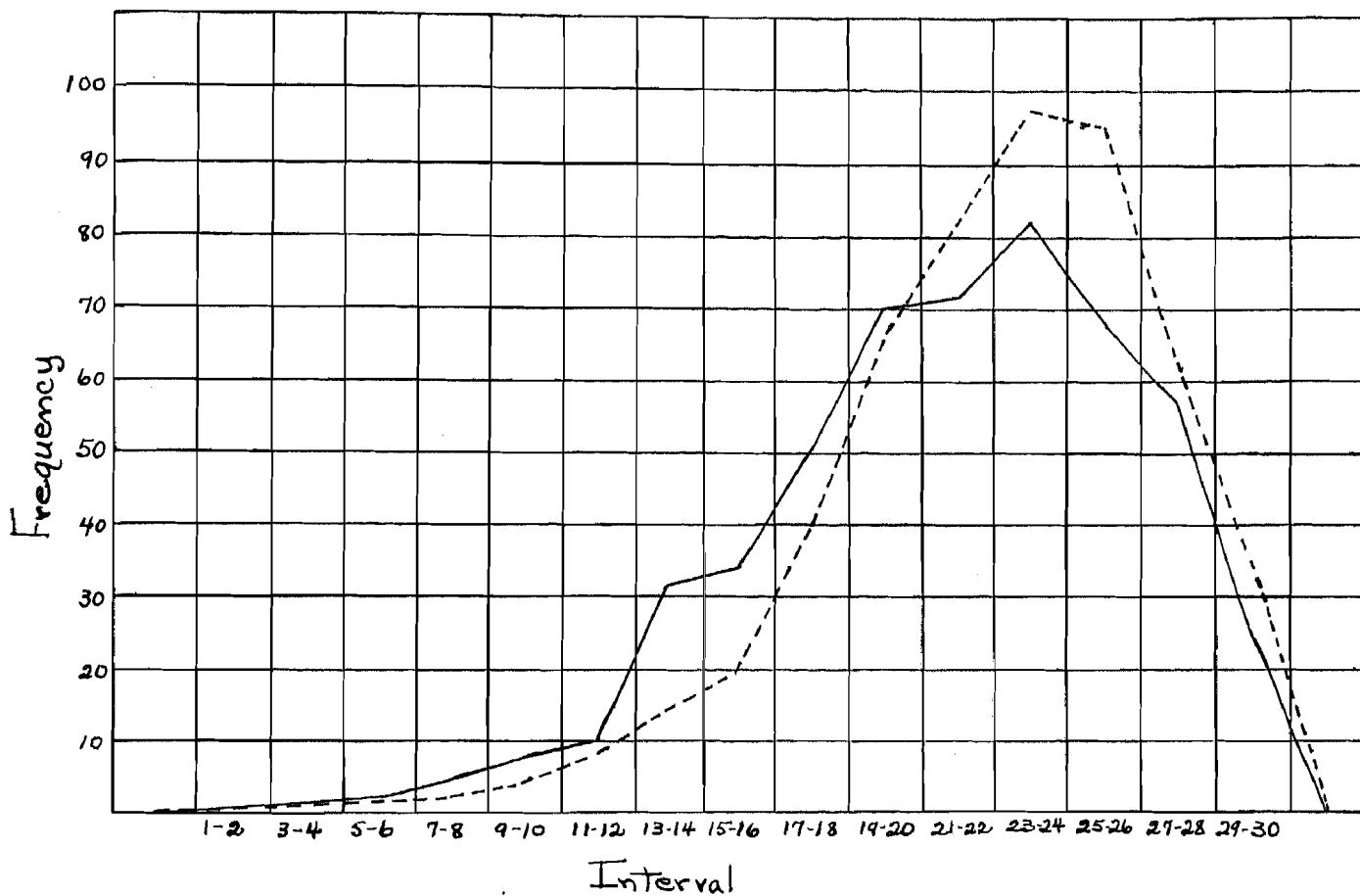


Figure b. Distribution of Scores on Part I, Forms A and B of Test II

Legend: — Form A - - - - - Form B

CHAPTER VI

SUMMARY AND CONCLUSIONS

This study consists of a description of the details of standardizing a series of tests in Plane Geometry and of an analysis of the results obtained from the construction and use of the tests.

The criteria for establishing the validity of the content of this test were six textbooks of geometry; the report of the Recommendations of the National Committee of Mathematics; the requirements of the College Entrance Examination Board; the Kansas State Course of Study; and criticisms of teachers and supervisors of mathematics, and of test construction specialists. Since the content of the test is in agreement with these reputable criteria, a high degree of validity has been assured.

The tests of this study are thoroughly objective and are unadulterated by factors which represent the psychological reactions of the teachers, they measure the pupils ability and attainment in geometry.

The tests give the student a sampling of practically all types of work covered in geometry; they insure further that all pupils have a fair chance to make a creditable showing according to their ability; and they tend to provide a high reliability of measurement.

The reliability coefficients are sufficiently high to make the tests valuable instruments for measuring class and individual attainment. Since the majority of students ranked in the same quartile or deviated from perfect consistency by only one quartile, it is concluded the tests possess a high degree of reliability.

The diversity from item to item in the degree of accuracy of the responses tends to show that the test is valid because, since none of the questions is answered correctly or incorrectly by all, it is evident that the test differentiates among students at both ends of the distributions.

An analysis of the results leads to the suggestion that geometry teachers should give greater attention to the fundamentals in the subject and that they should require a greater mastery of these fundamentals.

Percentiles differences on each form of the test could be due to practice effect on test mechanics or intervening study; or perhaps a discussion of Form A had taken place before giving Form B.

The graphical representation of the distribution of scores on the Tests follow in general the outline of a symmetrical bell-shaped curve, or the normal curve.

The statistical findings of this study clearly indicate that the test is valid and reliable.

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