A COMPARISON OF THE SYNTHETIC AND ANALYTIC METHODS OF TEACHING PLANE GEOMETRY.

A THESIS,

SUBMITTED TO THE DEPARTMENT OF EDUCATION AND THE GRADUATE COUNCIL OF THE KANSAS STATE TEACHERS COLLEGE OF EMPORIA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

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JULY 1933

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TABLE OF CONTENTS

A COMPARISON OF THE SYNTHETIC AND ANALYTIC METHODS OF TEACHING PLANE GEOMETRY.

LIST OF TABLES

Table:

LIST OF TABLES--Continued

LIST OF GRAPHS

Graph:

CHAPTER I

INTRODUCTION

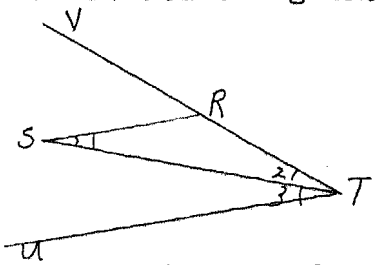
This study is concerned with the relative merits of the synthetic and analytic methods of teaching plane geometry.

Of the two methods, the synthetic method is the one commonly used. Only a few textbook writers mention the analytic method and then only in relation to some very difficult problem which usually is in the latter half of the book. It seems rather queer to the writer why the analytic method is used in case of some difficult exercise and then ignored in case of the simpler exercises.

The features of the synthetic method may be brought out by a simple example. In the proof, the synthetic method begins with the hypothesis and through a series of dependent steps proceeds to the conclusion, as in the following example:

Hypothesis: ST bisects ∠ VTU
SR // UT

Prove: △SRT is isosceles



Proof: The pupil using the synthetic method of attack would make his proof like the following:

$\angle 2 = \angle 3$	Reason: It is given that ST bisects $\angle VTU$.
$\angle 1 = \angle 3$	Reason: If two parallel lines are cut by a transversal, the alternate interior angles are equal.
$\therefore \angle 1 = \angle 2$	Reason: Quantities equal to the same quantity are equal to each other.
$\therefore$ SR = TR	Reason: If two angles of a triangle are equal, the sides opposite the two equal angles are equal.
$\therefore \triangle$SRT is isosceles	Reason: If two sides of a triangle are equal, the triangle is isosceles.

The pupil using the analytic method of attack would begin with the conclusion and work back to the hypothesis in order to discover a way to solve the problem; then he would reverse his steps and proceed as in the synthetic method. The following would be his method of attack of the above problem:

$\triangle$SRT is isosceles if SR = TR	Reason: If a triangle has two equal sides, it is isosceles.
SR = TR if $\angle 1 = \angle 2$	Reason: If two angles of a triangle are equal, the sides opposite them are equal.
$\angle 1 = \angle 2$ if $\angle 1 = \angle 3$ and $\angle 2 = \angle 3$	Reason: Quantities equal to the same quantity are equal to each other.
but $\angle 1 = \angle 3$	Reason: If two parallel lines are cut by a transversal, the alternate interior angles are equal.
and $\angle 2 = \angle 3$	Reason: It is given that ST bisects $\angle VTU$.

The pupil then reverses his steps and proceeds as in the synthetic method. He now has a logical proof which was developed through a series of steps to which he was led by the analysis.

Thus it may be seen that the synthetic method begins with the hypothesis and proceeds by means of a series of dependent steps to the conclusion; while the analytic method begins with the conclusion and works back to the hypothesis in order to find a way to solve the problem; and then proceeds as in the synthetic method. In short, the purpose of the analytic method is to remove the element of chance in developing a proof for a geometric exercise.

There seems to be very little literature of an authoritative nature upon the subject of the synthetic and analytic methods of teaching plane geometry.

Barnes[1] says:

> I have not measured in a statistical way the results of this study by analysis as compared with the usual synthetic work, but I am sure of several things from observation:
>
> 1. Pupils like to demonstrate when they feel they are doing a large part of the work constructively themselves.
>
> 2. Pupils have more initiative and are more independent since the procedure keeps a very definite problem before them and they are not merely acquiescing in statements of a text.
>
> 3. Pupils are far more efficient in demonstration of originals, for the simple reason they are familiar with the true method of attack.
>
> 4. Pupils carry this analysis into other subjects....

1. Barnes, H. O., Geometry by Analysis, SCHOOL REVIEW, October 1919

Pitts and Davis[1] made a comparison of the analytic and synthetic methods of teaching geometry. They used the special form paper advocated by Barnes.[2] Their results were adverse to the analytic method, but there were some questionable elements which entered into their study. First, different teachers taught the classes; second, the pupils were not paired; third, they used the analytic method for one semester only and then changed to the synthetic method. According to Schlauch[3] the analytic method will not secure superior results until the second semester. Fourth, they used the cumbersome form paper advocated by Barnes.[4]

Pitts and Davis have the following comment in their summary:

> The mechanics of the form paper were so difficult that the slower pupils could not understand the system to use it and it was so cumbersome that it interfered with the thinking of the very bright pupils. The details of filling the proper spaces with correct majors and minors and authorities were so tedious that pupils lost sight of the demonstration and spent more time deciding in which column a statement should appear than in arranging a logical solution.

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1. Pitts, Lemuel and Davis, Robert A., "A comparison of the Analytic and Synthetic Methods of Teaching Geometry", SCHOOL SCIENCE AND MATHEMATICS, Vol. 31, 1931.

2. Ibid. Page 3.

3. Schlauch, W. S., "The Analytic Method in Teaching of Geometry," NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, FIFTH YEARBOOK.

4. Ibid. Page 3.

Schlauch[1] has the following comment to make concerning the analytic method:

> Experience in teaching geometry by the analytic method and study of the results of numerous tests of the pupil's ability to analyze, extending over a period of twenty years in the High School of Commerce, New York City, have convinced me of the following facts:
>
> 1. Adequate mastery with accompanying pleasure and a thirst for original exercises comes in the second semester. A school year should be devoted to the subject. (Pitts and Davis[2] used only one semester for their study.)
>
> 2. The pupil needs training in analysis applied to the various types of geometric subject matter found in the conventional five books of plane geometry.
>
> 3. Teaching the method of analysis is difficult in the early stages.....
>
> 4. To master plane geometry, using the analytic method of attack, requires about a year for the majority of pupils.

Of the above comments by the different writers, it seems reasonable to expect that those of Schlauch are probably the most valid due to his many years of experience. However his conclusions are based on observations rather than from the results of a controlled experiment.

It is the purpose of this study to make a comparison of the synthetic and analytic methods of teaching plane geometry by means of a controlled experiment.

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1. Ibid. Page 4.

2. Ibid. Page 4.

CHAPTER II

METHOD OF PROCEDURE

The data for this study were secured by means of an experiment conducted during the school year of nineteen hundred thirty-two and thirty-three. The beginning geometry students of the Gallup, New Mexico, Senior High School were divided into two groups of approximately equal ability. One of these groups was taught theorems, constructions, loci, and the solution of original exercises by means of the analytic method. The analytic method assumes the problem solved and works back to some known truth. The other group was taught theorems, constructions, loci, and the solution of original exercises by means of the synthetic method. The synthetic method begins with a known fact and works towards the conclusion. The analytic method group met during the fifth period and the synthetic group met during the sixth period. Both groups met in the same room with the writer as instructor. The same text[1] was used by both groups, and the same tests were given each group on the same day. With the exception of theorems, constructions, loci, and the solution of original exercises, both groups were taught in the same manner. The original

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1. Hawkes, Luby, Touton. New Plane Geometry. Ginn and Company 1931

enrollment of the analytic method group was twenty-seven and that of the synthetic method group was twenty-nine. This enrollment varied somewhat during the school year. The length of the class period for each class was sixty minutes of which one-half was used for recitation and one half for supervised study.

From the two groups, sixteen pairs were selected. These students were paired according to their intelligence quotient, sex, age, nationality, and their ninth grade record. In order for two students to be paired they had to have almost the same intelligence quotient, to be of the same sex, and to differ very little in their ninth grade record. Only two pairs had an age difference of more than nine months. Like nationalities were paired in all possible cases. The intelligence test used was the Army Alpha[1] Test, Form V. This test was given at the beginning of the school year to both groups on the same day. Twenty-nine of the thirty-two pupils used in this study were graduates of the Gallup Junior High School. Their ninth grade records were secured from the office of the principal, while the records of the remaining three were secured from their transcripts. The algebra grade was not considered. Due to the different elements entering into the pairing and to the high correlation required, only sixteen pairs were obtained

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1. Published by the Bureau of Measurements, Kansas State Teachers College, Emporia, Kansas.

from a total of fifty-six pupils. Yet due to the care used in pairing, similar classroom conditions, and to the length of the experiment--thirty-six weeks--it is reasonable to expect that the results should be quite reliable and valid enough upon which to base conclusions. Reliability and validity are essential elements in a thesis of this type.

There were eight tests, exclusive of the final, given during the school year at intervals of from four to six weeks. Five of the tests were the five parts of the Lane-Greene Unit-Achievement Test.[1] The remaining three tests were devised by the author. At the end of the year a final test was given. This test was composed of four parts, and each part was given on successive days. The first part was a test on geometric facts such as, "What is the relation of the opposite angles of an inscribed quadrilateral". The second part was a test on definitions, such as, "What is the median of a triangle?" The third part consisted of a test on constructions and loci. The fourth part consisted of a test on the solution of original exercises. It is reasonable to expect that the analytic or synthetic methods would have little or no effect upon the results of Part I and II of the final test; while Parts III and IV are solved either by the analytic or synthetic method depending upon the group in which the pupil is enrolled. All materials for the final tests were secured from the text. All tests were carefully supervised, the superintendent of

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1. Published by Ginn and Company, Chicago, Illinois, 1929

schools and high school principal, both assisting during the final tests. A copy of each of the tests may be found in the appendix.

If the two groups have been paired very accurately, it is reasonable to expect that the results of those tests which do not involve constructions, loci, theorems, and the solution of original exercises, should be practically the same for both methods; while those tests that consider only the pupil's ability to solve original exercises, to make constructions, and find loci should differ as the efficiency of the two methods differ. These differences should enable us to compare the relative merits of the analytic and synthetic methods.

CHAPTER III

DATA

The purpose of this chapter is to present the data secured from the records and each of the tests. This presentation of data will be by means of tables and graphs.

Table I, on the following page, shows the manner in which the pupils were paired. The intelligence quotient and ninth grade records were obtained as explained in Chapter II. The ninth grade records were weighted as follows: an A grade was weighted as 5, B grade as 4, C grade as 3, D grade as 2, and an F grade as 1. In the table a grade of 3.6 means an average between C and B. The pairing is read across the page thus: of the two pupils of pair 1, the pupil in the synthetic method group has an intelligence quotient of 115 and a ninth grade record of 4.4, while the pupil in the analytic method group has an intelligence quotient of 113 and a ninth grade of 4.3.

The mean intelligence quotient of the synthetic method group was 99.9 while that of the analytic method group is 99.6--a difference of only 0.3 in favor of the synthetic method group. The ninth grade record of the synthetic method group showed a median of 3.46, while that of the analytic method group was 3.50--a difference of only 0.04 in favor of the analytic method group. This table also shows the means

TABLE I... PAIRING OF THE TWO GROUPS.

	SYNTHETIC		ANALYTIC	
Pair	I.Q.	9th grade record	I.Q.	9th grade record
1	115	4.4	113	4.3
2	104	3.9	106	3.9
3	102	3.9	108	3.9
4	108	4.3	108	4.0
5	100	3.4	98	3.8
6	109	3.4	107	3.6
7	91	3.5	94	3.3
8	109	3.3	108	3.6
9	100	4.4	100	3.9
10	103	3.8	100	3.4
11	93	3.8	95	3.8
12	97	3.0	91	3.6
13	86	3.0	84	3.2
14	97	3.4	102	3.4
15	90	2.8	90	3.0
16	95	3.0	89	3.6
Range	86-115	2.8-4.4	84-113	3.0-4.3
Mean	99.9	3.46	99.6	3.50
Mean upper 1/4 of class	110.25	4.25	109.25	4.03
Mean middle 1/2 of class	99.5	3.56	100.5	3.56
Mean lower 1/4 of class	90.0	2.95	88.5	3.23

Read table thus: Black indicates synthetic method; red indicates analytic method. The intelligence quotient for pupil of pair 1 of the synthetic method was 115, and his ninth grade record was 4.4. The intelligence quotient for pupil of pair 1 of the analytic method was 113, and his ninth grade record was 4.3. Read remainder of table accordingly.

for the upper one fourth, middle one half, and lower one fourth of the two groups. In each case it may be seen that the differences of the two groups are very slight. Figures pertaining to the synthetic method group are in black and those pertaining to the analytic method group are in red.

RESULTS OF TEST I

Table II on the following page shows the results of Test I. This test was Test 2 of the Lane-Greene Unit-Achievement Tests--a test on fundamental ideas of plane geometry. A copy of this test is in the appendix.

The results of this test were practically the same for the two divisions, and might be expected, for neither the analytic or synthetic method had been used up to this point. The mean for the synthetic method group was 16.6 and that for the analytic method group was 15.4 a difference of 1.2 in favor of the synthetic method group. For the upper one fourth, middle one half, and lower one fourth of the classes, the means of the synthetic method groups were slightly higher than the corresponding means of the analytic method groups. The ranges for both groups were almost identical--the range of the synthetic method group being 10-24 and that for the analytic method group 9-23.

Graph I is a comparison of the two groups. This graph shows the closeness of the results for the two classes.

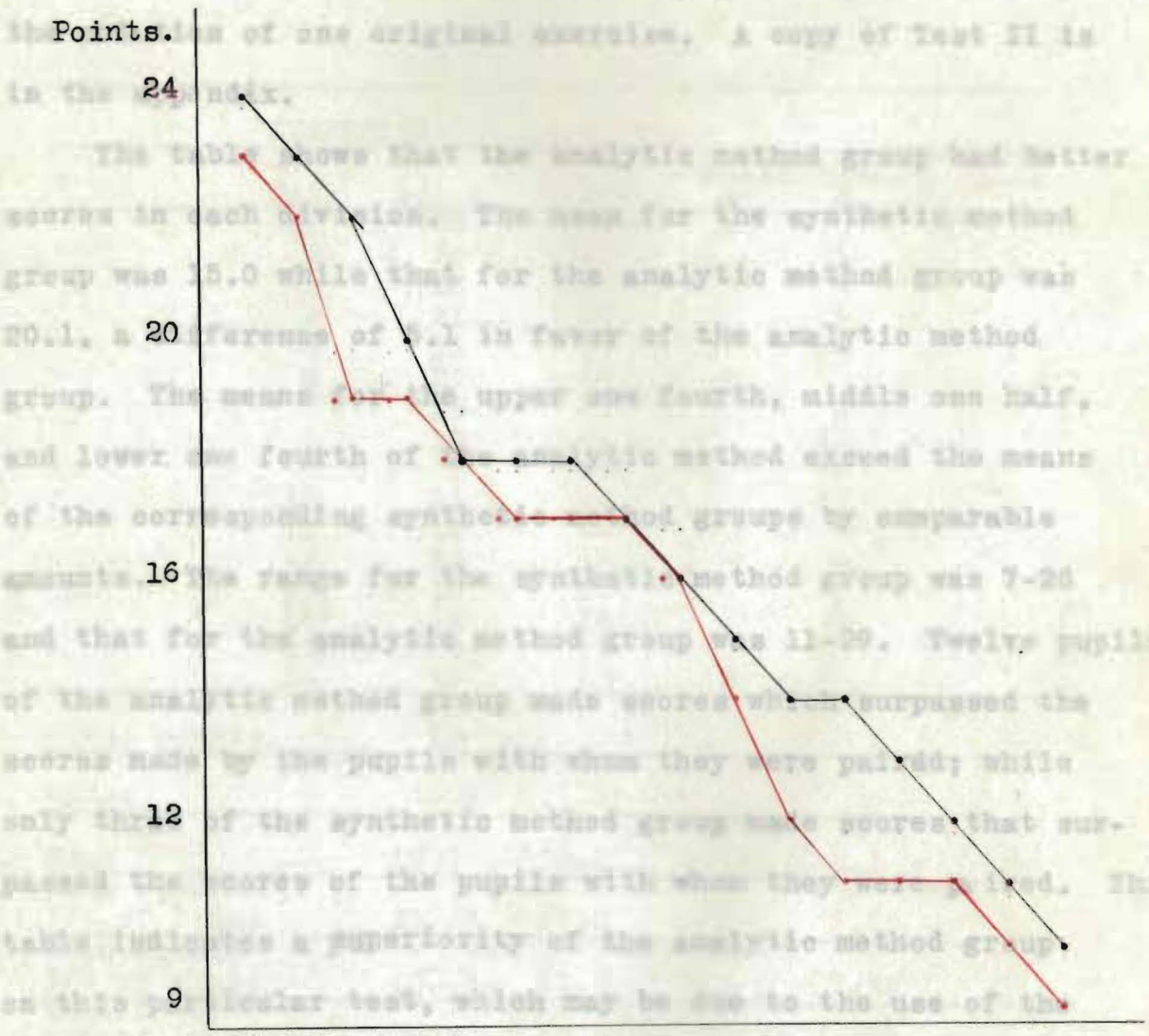
TABLE II... SHOWING RESULTS OF TEST I...TEST 1 OF THE LANE-GREENE UNIT-ACHIEVEMENT TESTS ON FUNDAMENTAL IDEAS OF GEOMETRY.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	22	18	4	
2	24	23	1	
3	18	19		1
4	23	19	4	
5	11	11	-	-
6	18	16	2	
7	16	17		1
8	17	11	6	
9	14	14	-	-
10	20	12	8	
11	18	17	1	
12	13	10	3	
13	15	11	4	
14	14	22		8
15	10	17		7
16	12	9	3	
Total points	34			
Range	10-24	9-23		
Mean	16.6	15.4	1.2	
Mean upper 1/4 of class	22.25	20.75	1.5	
Mean middle 1/2 of class	16.25	15.25	1.0	
Mean lower 1/4 of class	11.5	10.25	1.25	

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 22; for analytic method 18. The difference is four in favor of the synthetic method. Read remainder of table accordingly.

GRAPH I

SCORES OF TWO GROUPS ON TEST I



Read table thus: Black indicates synthetic method; red indicates analytic method. The highest score in the synthetic method group was 24, in the analytic method group 23. The scores are arranged in descending order.

RESULTS OF TEST II

Table III on the following page shows the results of Test II--a test on definitions, facts, constructions, and the solution of one original exercise. A copy of Test II is in the appendix.

The table shows that the analytic method group had better scores in each division. The mean for the synthetic method group was 15.0 while that for the analytic method group was 20.1, a difference of 5.1 in favor of the analytic method group. The means for the upper one fourth, middle one half, and lower one fourth of the analytic method exceed the means of the corresponding synthetic method groups by comparable amounts. The range for the synthetic method group was 7-26 and that for the analytic method group was 11-29. Twelve pupils of the analytic method group made scores which surpassed the scores made by the pupils with whom they were paired; while only three of the synthetic method group made scores that surpassed the scores of the pupils with whom they were paired. This table indicates a superiority of the analytic method group on this particular test, which may be due to the use of the analytic method; but more likely to the failure of the three pupils of the synthetic method in pairs 5, 8, and 9 to do correspondingly as well as the other pupils of their class.

Graph II gives a comparison of the results of this test for the analytic and synthetic method groups.

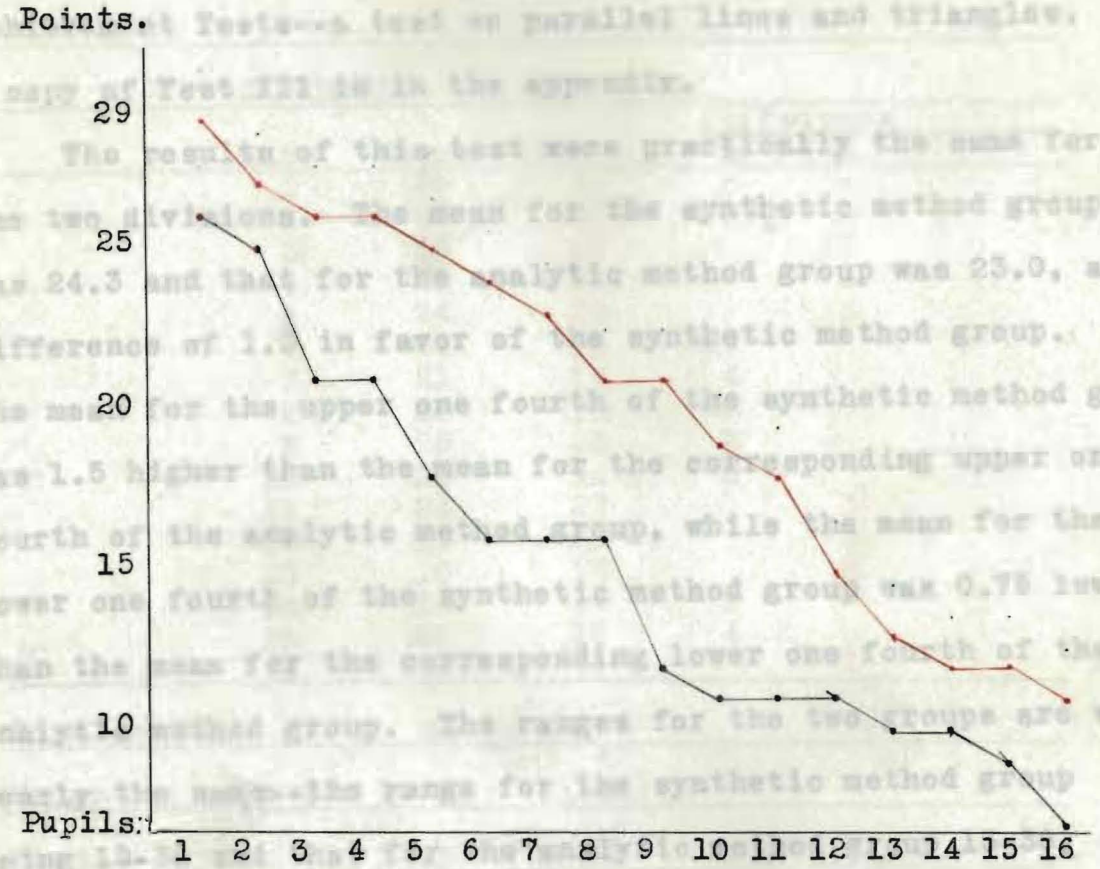
TABLE III... SHOWING RESULTS OF TEST II...A TEST ON DEFINITIONS, FACTS, AND CONTRUCTIONS.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	26	24	2	
2	25	26		1
3	16	25		9
4	21	27		6
5	11	29		18
6	21	19	2	
7	18	21		3
8	10	21		11
9	7	23		16
10	16	26		10
11	11	15		4
12	12	12	-	-
13	10	11		1
14	11	18		7
15	9	12		3
16	16	13	3	
Total points	30			
Range	7-26	11-29		
Mean	15.0	20.1		5.1
Mean upper 1/4 of class	23.25	27.0		3.75
Mean middle 1/2 of class	13.75	21.5		7.75
Mean lower 1/4 of class	9.0	12.0		3.0

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 26; for analytic method 24. The difference is 2 in favor of the synthetic method. Read remainder of table accordingly.

GRAPH II

SCORES OF THE TWO GROUPS ON TESTII



Read table thus: Black indicates synthetic method; red indicates analytic method. The highest score in the synthetic method was 26, in the analytic method 29. The scores for each of the pupils in the two method groups are arranged in descending order.

RESULTS OF TEST III

Table IV on the following page shows the results of Test III. This test was Test 3 of the Lane-Green Unit-Achievement Tests--a test on parallel lines and triangles. A copy of Test III in in the appendix.

The results of this test were practically the same for the two divisions. The mean for the synthetic method group was 24.3 and that for the analytic method group was 23.0, a difference of 1.3 in favor of the synthetic method group. The mean for the upper one fourth of the synthetic method group was 1.5 higher than the mean for the corresponding upper one fourth of the analytic method group, while the mean for the lower one fourth of the synthetic method group was 0.75 lower than the mean for the corresponding lower one fourth of the analytic method group. The ranges for the two groups are very nearly the same--the range for the synthetic method group being 10-38 and that for the analytic method group 13-38.

Graph III shows the comparison of the two groups. This graph indicates the closeness of the scores for the two groups. Neither the table nor the graph shows any significant difference for the two groups in the results of Test III.

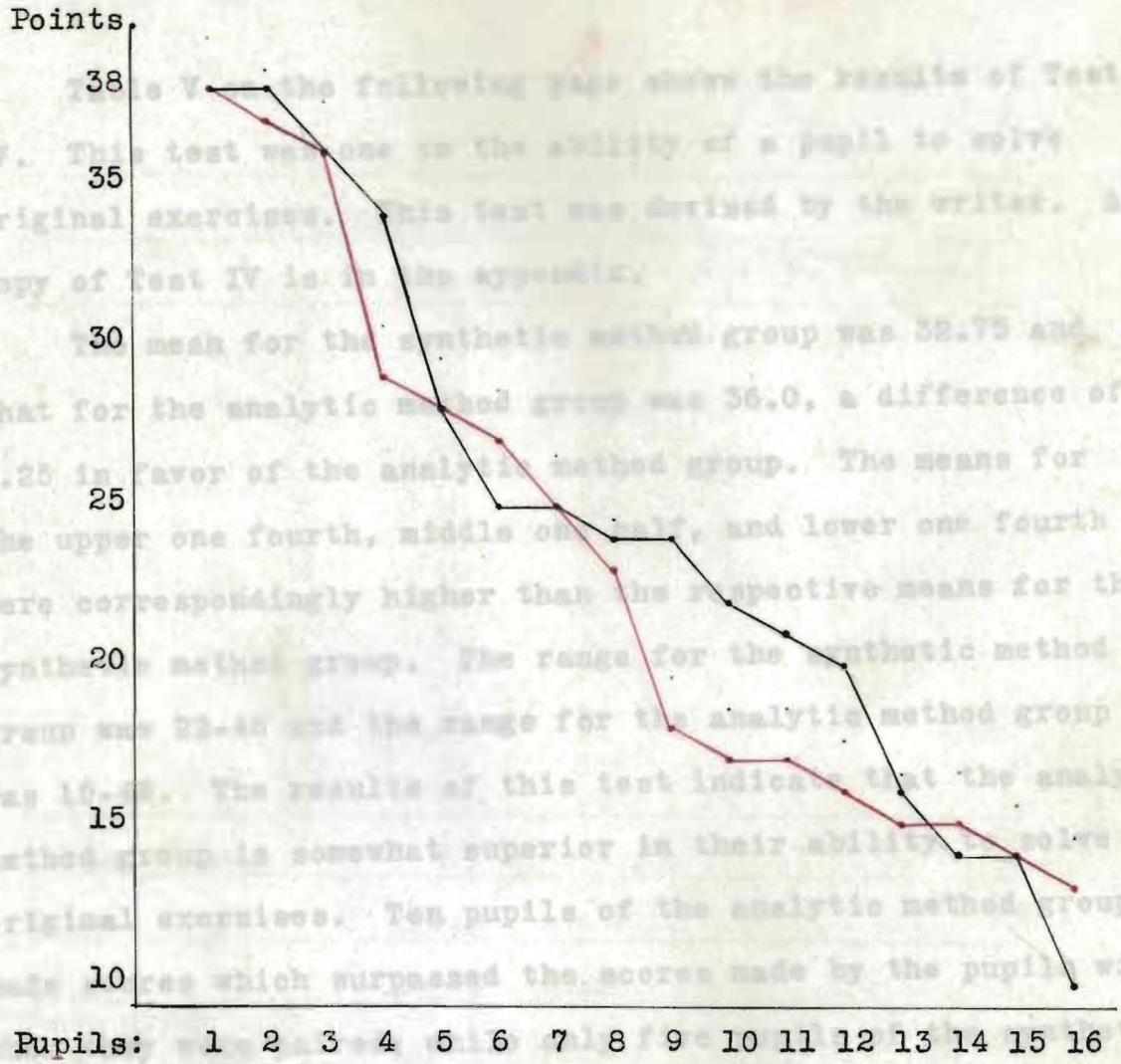
TABLE IV... SHOWING RESULTS OF TEST III...TEST 2 OF THE LANE-GREENE UNIT-ACHIEVEMENT TESTS ...A TEST ON PARALLEL LINES AND TRIANGLES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	36	36	-	-
2	38	38	-	-
3	25	37		12
4	28	28	-	-
5	24	29		5
6	34	15	19	
7	21	17	4	
8	38	15	23	
9	25	23	2	
10	22	14	8	
11	24	27		3
12	14	25		9
13	10	18		8
14	14	13	1	
15	20	16	4	
16	16	17		1
Total points	60			
Range	10-38	13-38		
Mean	24.3	23.0	1.3	
Mean upper 1/4 of class	36.5	35.0	1.5	
Mean middle 1/2 of class	23.6	21.4	2.2	
Mean lower 1/4 of class	13.5	14.25		.75

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair three for the synthetic method was 25; for analytic method 37. The difference is 12 in favor of the analytic method. Read remainder of table accordingly.

GRAPH III

SHOWING SCORES OF PUPILS ON TEST III



Read table thus: Black indicates the synthetic method; red indicates the analytic method. The score of pupil 1 of the synthetic method group was 38, and for the analytic method 38. The scores are arranged in descending order.

RESULTS OF TEST IV

Table V on the following page shows the results of Test IV. This test was one on the ability of a pupil to solve original exercises. This test was devised by the writer. A copy of Test IV is in the appendix.

The mean for the synthetic method group was 32.75 and that for the analytic method group was 36.0, a difference of 3.25 in favor of the analytic method group. The means for the upper one fourth, middle one half, and lower one fourth were correspondingly higher than the respective means for the synthetic method group. The range for the synthetic method group was 22-48 and the range for the analytic method group was 15-48. The results of this test indicate that the analytic method group is somewhat superior in their ability to solve original exercises. Ten pupils of the analytic method group made scores which surpassed the scores made by the pupils with whom they were paired; while only five pupils of the synthetic method group made scores which surpassed scores of the pupils with whom they were paired.

Graph IV is a comparison of the two groups. This graph shows the small difference in favor of the analytic method group.

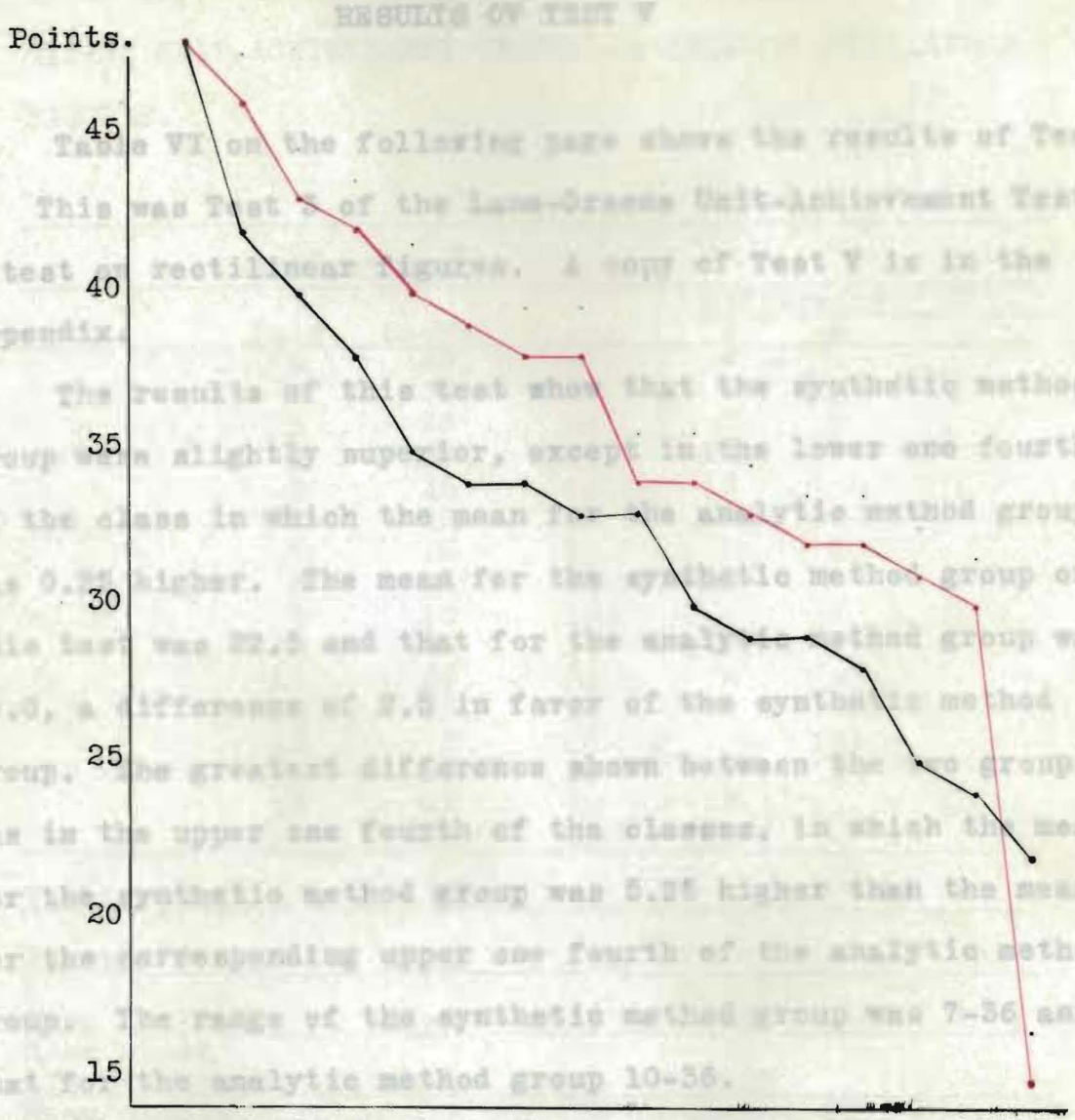
TABLE V... SHOWING RESULTS OF TEST IV...A TEST ON ORIGINAL EXERCISES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	29	48		19
2	35	42		7
3	38	39		1
4	48	46	2	
5	25	40		15
6	34	15	19	
7	30	32		2
8	33	33	-	-
9	40	38	2	
10	29	34		5
11	34	43		9
12	28	38		10
13	22	31		9
14	24	34		10
15	33	32	1	
16	42	30	12	
Total points	51			
Range	22-48	15-48		
Mean	32.75	36.0		3.25
Mean upper 1/4 of class	42.00	44.75		2.75
Mean middle 1/2 of class	32.10	36.0		3.9
Mean lower 1/4 of class	24.75	27.0		2.25

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 29; for analytic method 48. The difference is 19 in favor of the analytic method. Read remainder of table accordingly.

GRAPH IV

SHOWING SCORES OF PUPILS ON TEST IV



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 of the synthetic method group was 48, and that for pupil 1 of the analytic method group 48. The scores are arranged in descending order.

RESULTS OF TEST V

Table VI on the following page shows the results of Test V. This was Test 3 of the Lane-Greene Unit-Achievement Tests--a test on rectilinear figures. A copy of Test V is in the appendix.

The results of this test whow that the synthetic method group were slightly superior, except in the lower one fourth of the class in which the mean for the analytic method group was 0.25 higher. The mean for the synthetic method group on this test was 22.5 and that for the analytic method group was 20.0, a difference of 2.5 in favor of the synthetic method group. The greatest difference shown between the two groups was in the upper one fourth of the classes, in which the mean for the synthetic method group was 5.25 higher than the mean for the corresponding upper one fourth of the analytic method group. The range of the synthetic method group was 7-36 and that for the analytic method group 10-36.

Graph V is a comparison of the two groups. This graph indicates the superiority of the synthetic method group in the upper one fourth of the classes.

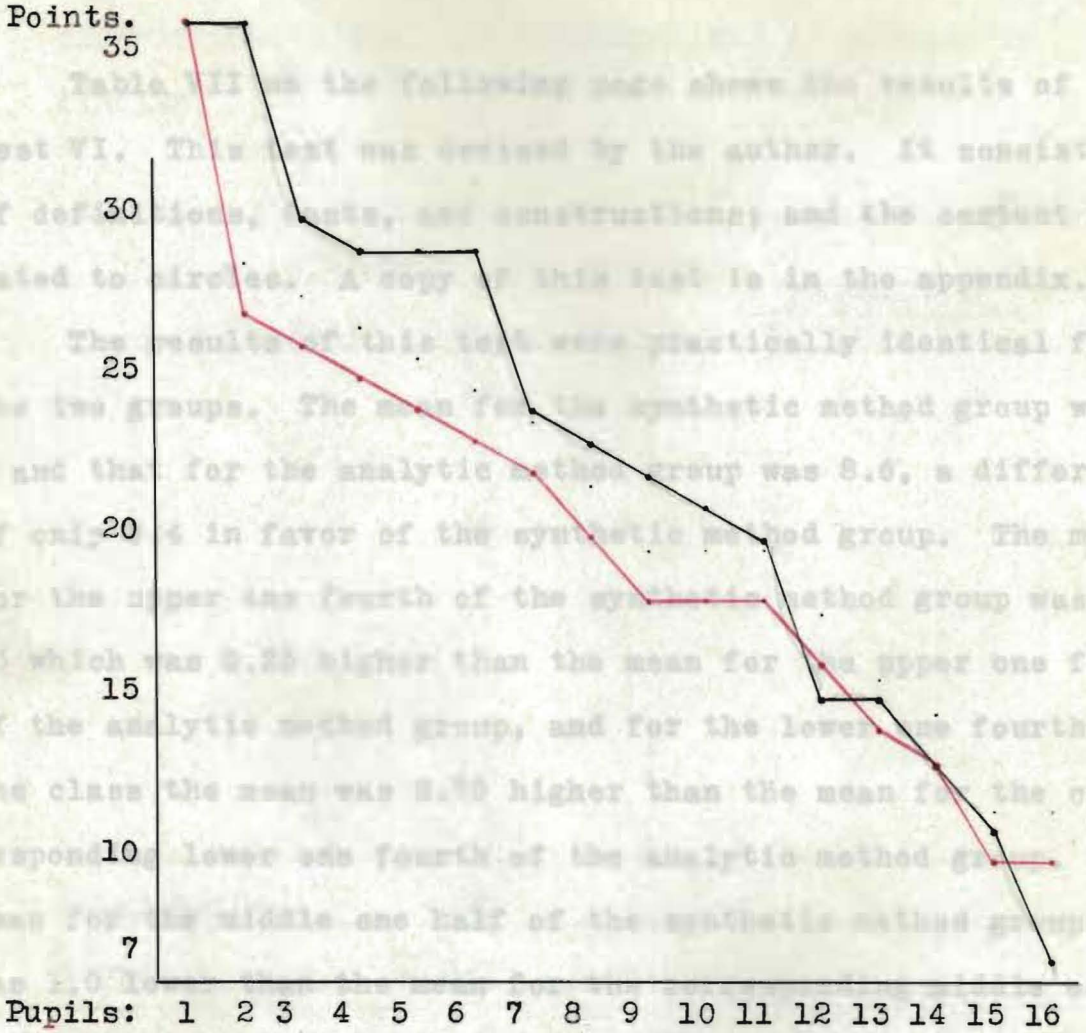
TABLE VI... SHOWING RESULTS OF TEST V...TEST 3 OF THE LANE-GREENE UNIT-ACHIEVEMENT TESTS...A TEST ON RECTILINEAR FIGURES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	29	25	4	
2	20	27		7
3	29	36		7
4	30	16	14	
5	36	18	18	
6	21	23		2
7	13	24		9
8	36	18	18	
9	11	18		7
10	24	14	10	
11	22	22	-	-
12	15	10	5	
13	7	26		19
14	15	20		5
15	29	10	19	
16	23	13	10	
Total points	59			
Range	7-36	10-36		
Mean	22.5	20.0	2.5	
Mean upper 1/4 of class	33.75	28.5	5.25	
Mean middle 1/2 of class	22.9	19.9	3.0	
Mean lower 1/4 of class	11.5	11.75		.25

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 29; for analytic method 25. The difference is 4 in favor of the synthetic method. Read remainder of table accordingly.

GRAPH V

SHOWING SCORES OF PUPILS ON TEST V



Read table thus: Black indicates the synthetic method group; red indicates the analytic method group. The score of pupil 1 in the synthetic method group was 36, and the score of pupil 1 in the analytic method group 36. The remainder of the scores are arranged in descending order.

RESULTS OF TEST VI

Table VII on the following page shows the results of Test VI. This test was devised by the author. It consisted of definitions, facts, and constructions; and the content related to circles. A copy of this test is in the appendix.

The results of this test were practically identical for the two groups. The mean for the synthetic method group was 9 and that for the analytic method group was 8.6, a difference of only 0.4 in favor of the synthetic method group. The mean for the upper one fourth of the synthetic method group was 15 which was 0.25 higher than the mean for the upper one fourth of the analytic method group, and for the lower one fourth of the class the mean was 0.75 higher than the mean for the corresponding lower one fourth of the analytic method group. The mean for the middle one half of the synthetic method group was 1.0 lower than the mean for the corresponding middle one half of the analytic method group. The range for the synthetic method group was 3-19 and that for the analytic method group 1-17.

Graph VI shows the comparison of the two groups and indicates the slight superiority of the analytic method group in the middle one half of the classes.

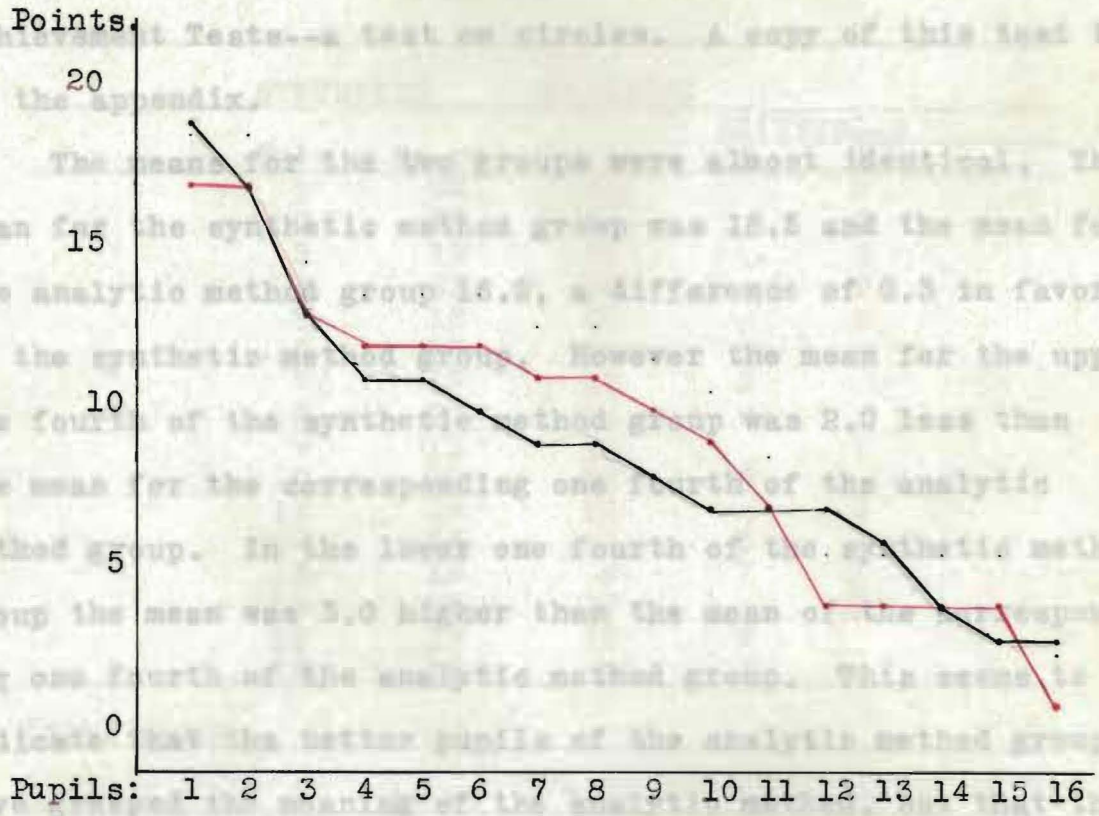
TABLE VII... SHOWING RESULTS OF TEST VI...A TEST ON DEFINITIONS, FACTS, AND CONSTRUCTIONS AS RELATED TO CIRCLES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	17	12	5	
2	19	17	2	
3	11	12		1
4	7	7	-	-
5	9	9	-	-
6	10	10	-	-
7	4	1	3	
8	3	4		1
9	11	11	-	-
10	3	13		10
11	13	11	2	
12	8	12		4
13	6	4	2	
14	7	4	3	
15	9	17		8
16	7	4	3	
Total points	20			
Range	3-19	1-17		
Mean	9.0	8.6	.4	
Mean upper 1/4 of class	15.0	14.75	.25	
Mean middle 1/2 of class	8.5	9.5		1.0
Mean lower 1/4 of class	4.0	3.25	.75	

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair 1 for the synthetic method was 19, for analytic method 17. The difference was 2 in favor of the synthetic method. Read remainder of table accordingly.

GRAPH VI

SHOWING SCORES OF PUPILS ON TEST VI



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 in the synthetic method group was 19, and the score of pupil 1 in the analytic method group 17. The remainder of the scores are arranged in descending order.

RESULTS OF TEST VII

Table VIII on the following page shows the results of Test VII. This test was Test 4 of the Lane-Greene Unit-Achievement Tests--a test on circles. A copy of this test is in the appendix.

The means for the two groups were almost identical. The mean for the synthetic method group was 18.5 and the mean for the analytic method group 18.2, a difference of 0.3 in favor of the synthetic method group. However the mean for the upper one fourth of the synthetic method group was 2.0 less than the mean for the corresponding one fourth of the analytic method group. In the lower one fourth of the synthetic method group the mean was 3.0 higher than the mean of the corresponding one fourth of the analytic method group. This seems to indicate that the better pupils of the analytic method group have grasped the meaning of the analytic method, but that the duller pupils have not. The range for the synthetic method group was 11-28 and that for the analytic method group 6-31.

Graph VII shows the comparison of the two groups and indicates the differences in the upper and lower one fourth of the two classes.

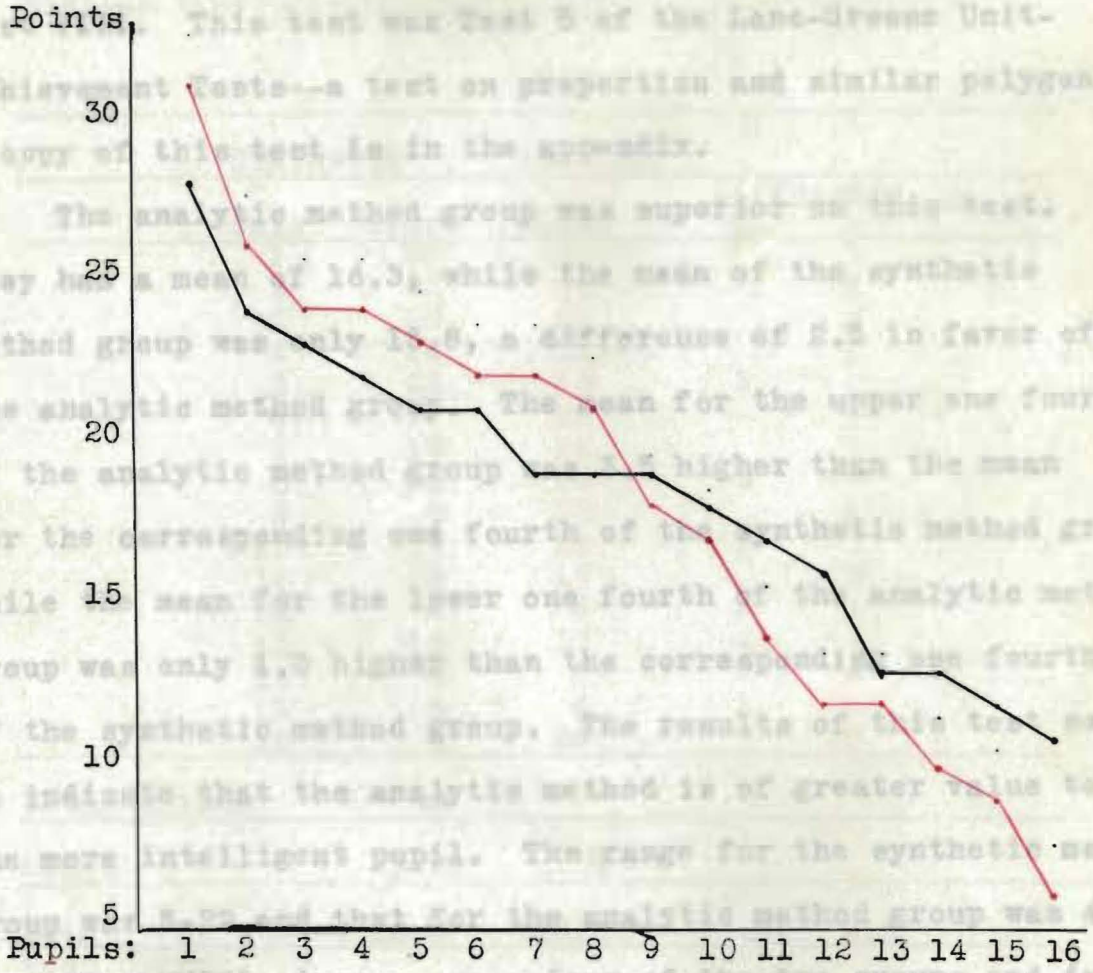
TABLE VIII... SHOWING RESULTS OF TEST VII...TEST 4 OF THE LANE-GREENE UNIT-ACHIEVEMENT TESTS... A TEST ON CIRCLES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	22	24		2
2	28	31		3
3	19	22		3
4	21	23		2
5	18	26		8
6	17	21		4
7	19	17	2	
8	23	14	9	
9	11	18		7
10	16	12	4	
11	24	24	-	-
12	13	10	3	
13	13	6	7	
14	21	22		1
15	19	12	7	
16	12	9	3	
Total points	45			
Range	11-28	6-31		
Mean	18.5	18.2	.3	
Mean upper 1/4 of class	24.25	26.25		2.0
Mean middle 1/2 of class	18.75	18.6	.15	
Mean lower 1/4 of class	12.25	9.25	3.0	

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 22; for analytic method 24. The difference is 2 in favor of the analytic method. Read remainder of table accordingly.

GRAPH VII

SHOWING SCORES OF PUPILS ON TEST VII



Read table thus. Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 in the synthetic method group was 28, and the score for pupil 1 in the analytic method group 31. The scores are arranged in descending order.

RESULTS OF TEST VIII

Table IX on the following page shows the results of Test VIII. This test was Test 5 of the Lane-Greene Unit-Achievement Tests--a test on proportion and similar polygons. A copy of this test is in the appendix.

The analytic method group was superior on this test. They had a mean of 16.3, while the mean of the synthetic method group was only 13.8, a difference of 2.5 in favor of the analytic method group. The mean for the upper one fourth of the analytic method group was 3.5 higher than the mean for the corresponding one fourth of the synthetic method group; while the mean for the lower one fourth of the analytic method group was only 1.0 higher than the corresponding one fourth of the synthetic method group. The results of this test seem to indicate that the analytic method is of greater value to the more intelligent pupil. The range for the synthetic method group was 5-22 and that for the analytic method group was 4-26.

Graph VIII gives a comparison of the two groups and indicates the superiority of the upper one fourth of the analytic method class as compared with the upper one fourth of the synthetic method class.

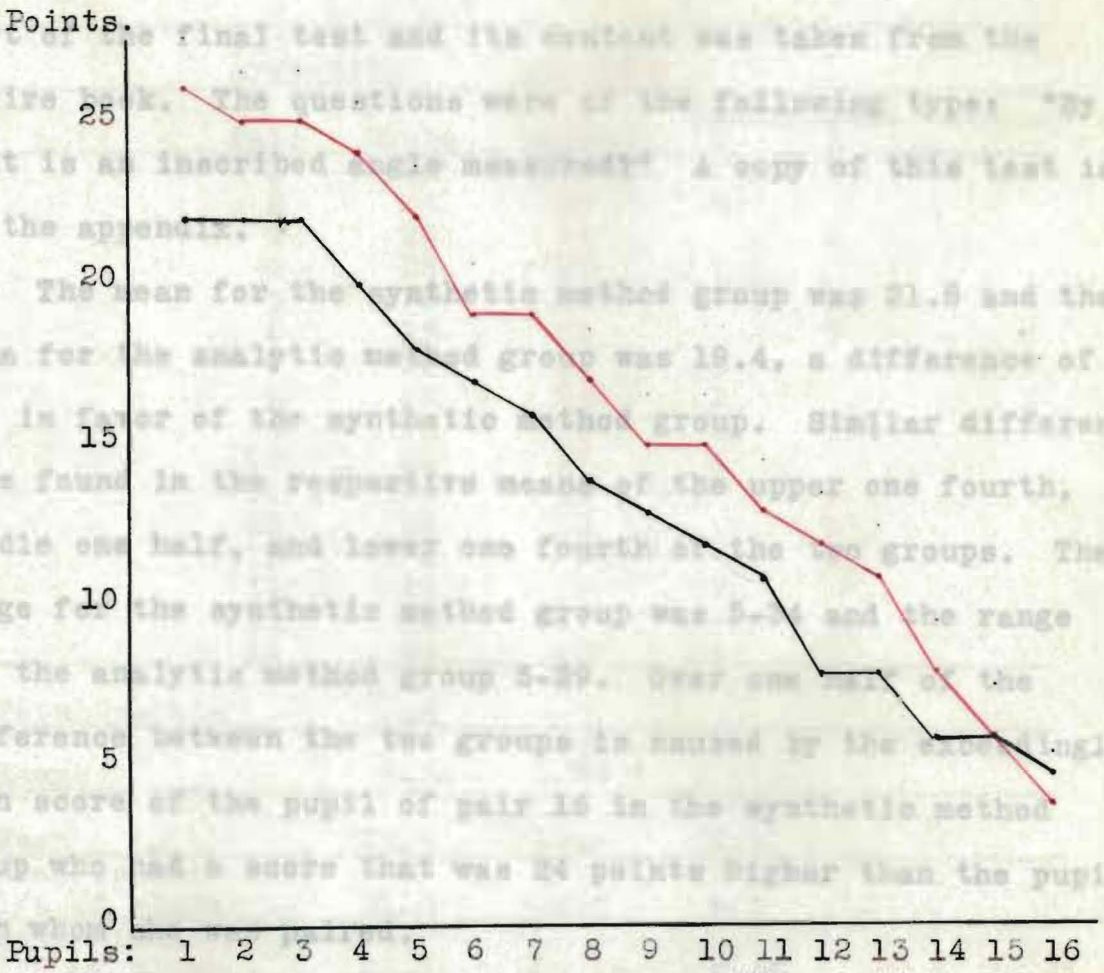
TABLE IX... SHOWING RESULTS OF TEST VIII...TEST 5 OF THE LANE-GREENE UNIT-ACHIEVEMENT TESTS...A TEST ON PROPORTION AND SIMILAR POLYGONS.

	SYNTHETIC	ANALYTIC		
			Difference	
Pair	Score	Score	S	A
1	22	25		3
2	22	25		3
3	20	24		4
4	16	17		1
5	22	19	3	
6	17	22		5
7	12	15		3
8	18	15	3	
9	6	12		6
10	14	11	3	
11	11	19		8
12	6	6	-	-
13	13	13	-	-
14	8	8	-	-
15	5	4	1	
16	8	26		18
Total points	45			
Range	5-22	4-26		
Mean	13.8	16.3		2.5
Mean upper 1/4 of class	21.5	25.0		3.5
Mean middle 1/2 of class	13.6	16.5		2.9
Mean lower 1/4 of class	6.25	7.25		1.0

Read table thus: Black indicates synthetic methos; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 22; for analytic method 25. The difference is 3 in favor of the analytic method. Read remainder of table accordingly.

GRAPH VIII

SHOWING SCORES OF PUPILS ON TEST VIII



Read table thus: Black indicates synthetic method groug; red indicates analytic method group. The score of pupil 1 of the synthetic method group was 22, and the score of pupil 1 of the analytic method group 26. The scores are arranged in descending order.

RESULTS OF TEST IX

Table X on the following page shows the results of Test IX--a test on geometric facts. This test was the first part of the final test and its content was taken from the entire book. The questions were of the following type: "By what is an inscribed angle measured?" A copy of this test is in the appendix.

The mean for the synthetic method group was 21.8 and the mean for the analytic method group was 19.4, a difference of 2.4 in favor of the synthetic method group. Similar differences were found in the respective means of the upper one fourth, middle one half, and lower one fourth of the two groups. The range for the synthetic method group was 5-34 and the range for the analytic method group 5-29. Over one half of the difference between the two groups is caused by the exceedingly high score of the pupil of pair 16 in the synthetic method group who had a score that was 24 points higher than the pupil with whom she was paired.

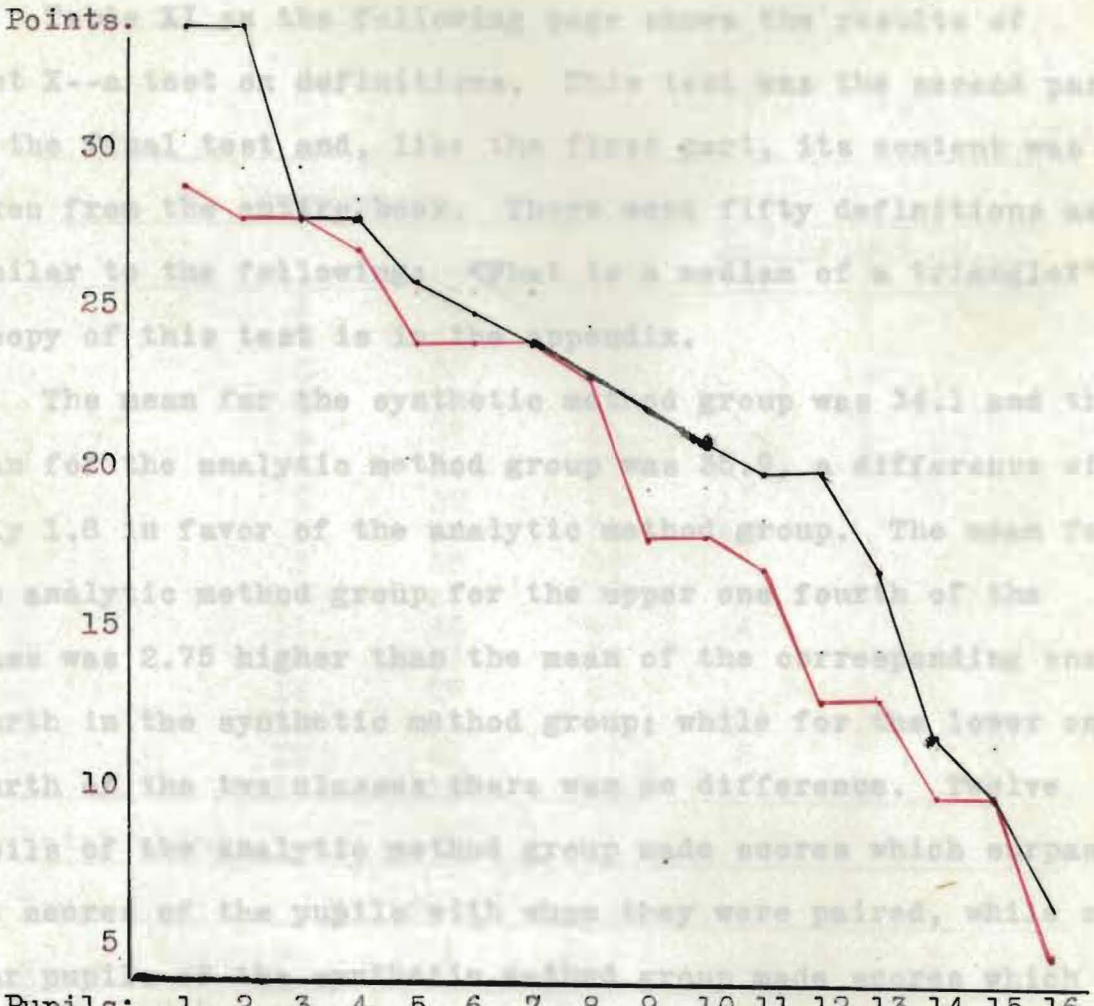
Graph IX gives a comparison of the two groups. This shows a small difference in favor of the synthetic method group.

TABLE X... SHOWING RESULTS OF TEST IX...A TEST ON GEOMETRIC FACTS.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	26	28		2
2	34	28	6	
3	20	24		4
4	28	27	1	
5	20	24		4
6	17	23		6
7	21	18	3	
8	25	24	1	
9	23	18	5	
10	22	17	5	
11	28	29		1
12	12	13		1
13	5	5	-	-
14	24	13	9	
15	10	10	-	-
16	34	10	24	
Total points	40			
Range	5-34	5-29		
Mean	21.8	19.4	2.4	
Mean upper 1/4 of class	31.0	28.0	3.0	
Mean middle 1/2 of class	22.6	20.25	2.35	
Mean lower 1/4 of class	11.0	9.5	1.5	

GRAPH IX

SHOWING SCORES OF PUPILS ON TEST IX



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 of the synthetic method group was 34, and the score of pupil 1 of the analytic method group 29. The scores are arranged in descending order.

RESULTS OF TEST X

Table XI on the following page shows the results of Test X--a test on definitions. This test was the second part of the final test and, like the first part, its content was taken from the entire book. There were fifty definitions asked similar to the following: "What is a median of a triangle?" A copy of this test is in the appendix.

The mean for the synthetic method group was 34.1 and the mean for the analytic method group was 35.9, a difference of only 1.8 in favor of the analytic method group. The mean for the analytic method group for the upper one fourth of the class was 2.75 higher than the mean of the corresponding one fourth in the synthetic method group; while for the lower one fourth of the two classes there was no difference. Twelve pupils of the analytic method group made scores which surpassed the scores of the pupils with whom they were paired, while only four pupils of the synthetic method group made scores which surpassed the scores of the pupils with whom they were paired. The range for the synthetic method group was 18-44 and the range for the analytic method group was 21-44. If we combine the results of Tests IX and X we find little difference between the two groups. This is reasonable to expect as the analytic or synthetic method does not enter directly into the teaching of facts and definitions.

Graph X shows the comparison of the two classes.

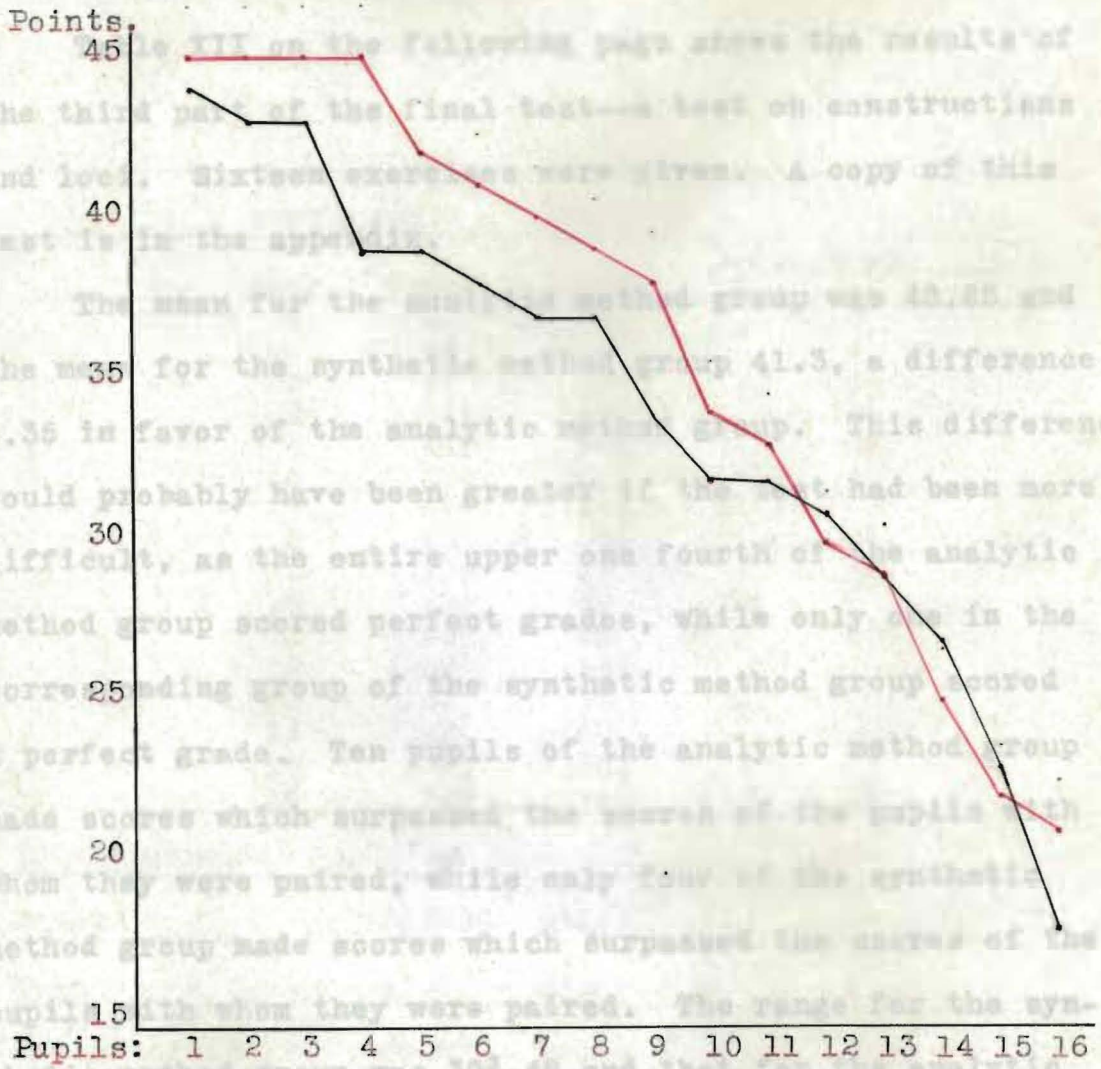
TABLE XI... SHOWING RESULTS OF TEST X...A TEST ON DEFINITIONS..

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	39	45		6
2	43	45		2
3	32	45		13
4	34	45		9
5	37	41		4
6	39	42		3
7	32	21	11	
8	43	33	10	
9	38	39		1
10	37	38		1
11	44	40	4	
12	23	29		6
13	29	30		1
14	31	34		3
15	27	25	2	
16	18	22		4
Total points	50			
Range	18-44	21-45		
Mean	34.1	35.9		1.8
Mean upper 1/4 of class	42.25	45.0		2.75
Mean middle 1/2 of class	35.0	38.1		3.1
Mean lower 1/4 of class	24.25	24.25	-	-

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 39; for analytic method 45. The difference is 6 in favor of the analytic method. Read remainder of table accordingly.

GRAPH X

SHOWING SCORES OF PUPILS ON TEST X



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 of the synthetic method group was 44, and the score of pupil 1 of the analytic method group 45. The scores are arranged in descending order.

RESULTS OF TEST XI

Table XII on the following page shows the results of the third part of the final test--a test on constructions and loci. Sixteen exercises were given. A copy of this test is in the appendix.

The mean for the analytic method group was 43.65 and the mean for the synthetic method group 41.3, a difference of 2.35 in favor of the analytic method group. This difference would probably have been greater if the test had been more difficult, as the entire upper one fourth of the analytic method group scored perfect grades, while only one in the corresponding group of the synthetic method group scored a perfect grade. Ten pupils of the analytic method group made scores which surpassed the scores of the pupils with whom they were paired, while only four of the synthetic method group made scores which surpassed the scores of the pupils with whom they were paired. The range for the synthetic method group was $30\frac{3}{4}$-48 and that for the analytic method group 33-48. The results of this test seem to indicate that the analytic method is slightly better for teaching constructions.

Graph XI gives a comparison of the two classes and shows the slight superiority of the analytic method group on this test.

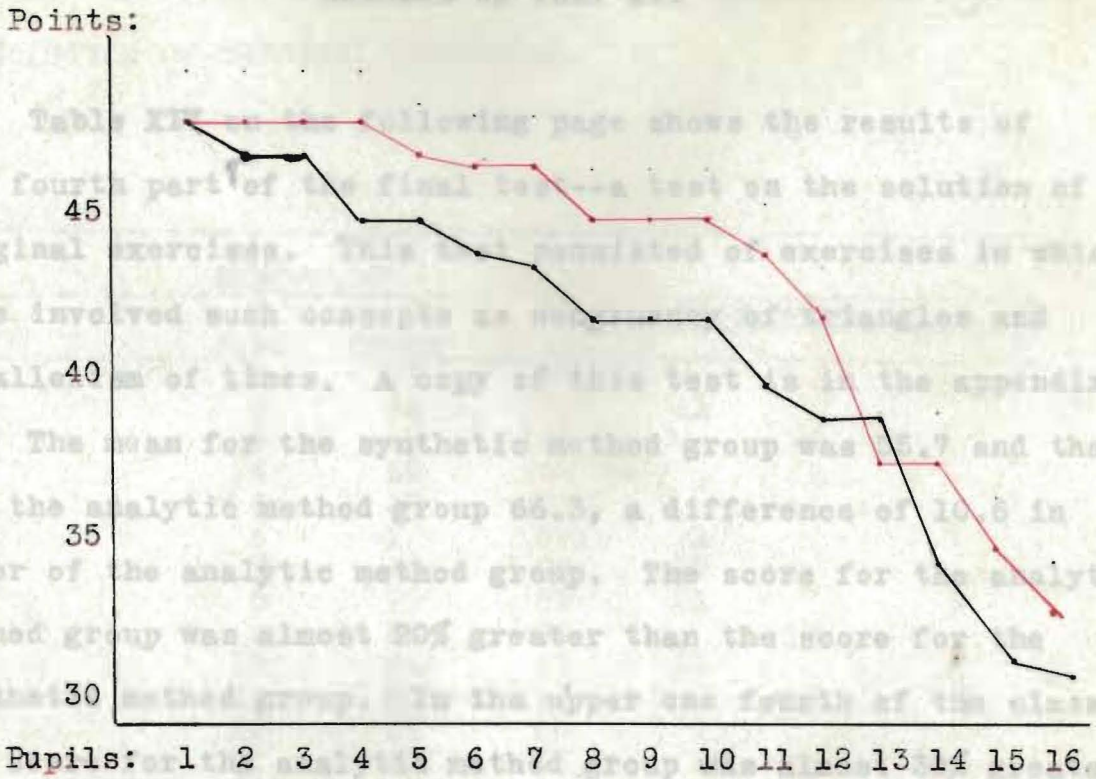
TABLE XII... SHOWING RESULTS OF TEST XI... A TEST ON CONSTRUCTIONS AND LOCI.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	45	48		3
2	48	48	-	-
3	42	46½		4½
4	31½	48		16½
5	44¼	46½		2¼
6	42	45		3
7	47¼	35¼	12	
8	45	47¼		2¼
9	39¾	45		5¼
10	47¼	45	2¼	
11	43½	48		4½
12	39	38¼	¾	
13	39	33	6	
14	42	42	-	-
15	30¾	38¼		7½
16	34½	44½		9½
Total points	48			
Range	30¾-48	33-48		
Mean	41.31	43.65		2.34
Mean upper 1/4 of class	46.89	48.00		1.11
Mean middle 1/2 of class	42.18	45.18		3.00
Mean lower 1/4 of class	33.93	36.18		2.25

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair 1 for the synthetic method was 45; for analytic method 48. The difference is 3 in favor of the analytic method. Read remainder of table accordingly.

GRAPH XI

SHOWING SCORES OF PUPILS ON TEST XI



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 of the synthetic method group was 48, and of pupil 1 of the analytic method group 48. The scores are in descending order.

RESULTS OF TEST XII

Table XIV on the following page shows the results of the fourth part of the final test--a test on the solution of original exercises. This test consisted of exercises in which were involved such concepts as congruency of triangles and parallelism of lines. A copy of this test is in the appendix.

The mean for the synthetic method group was 55.7 and that for the analytic method group 66.3, a difference of 10.6 in favor of the analytic method group. The score for the analytic method group was almost 20% greater than the score for the synthetic method group. In the upper one fourth of the class, the score for the analytic method group was almost 30% greater than that for the corresponding one fourth of the synthetic method group; while in the lower one fourth of the classes the results were almost identical. The results of this test indicate that the analytic method is superior for the average and above average pupil in the solution of original exercises, but that there is little difference between the two methods for the slower pupil. The range for the analytic method group was 26-112 and that for the synthetic method group was 16-94.

Graph XII shows the great difference between the two classes in the upper one fourth and middle one half.

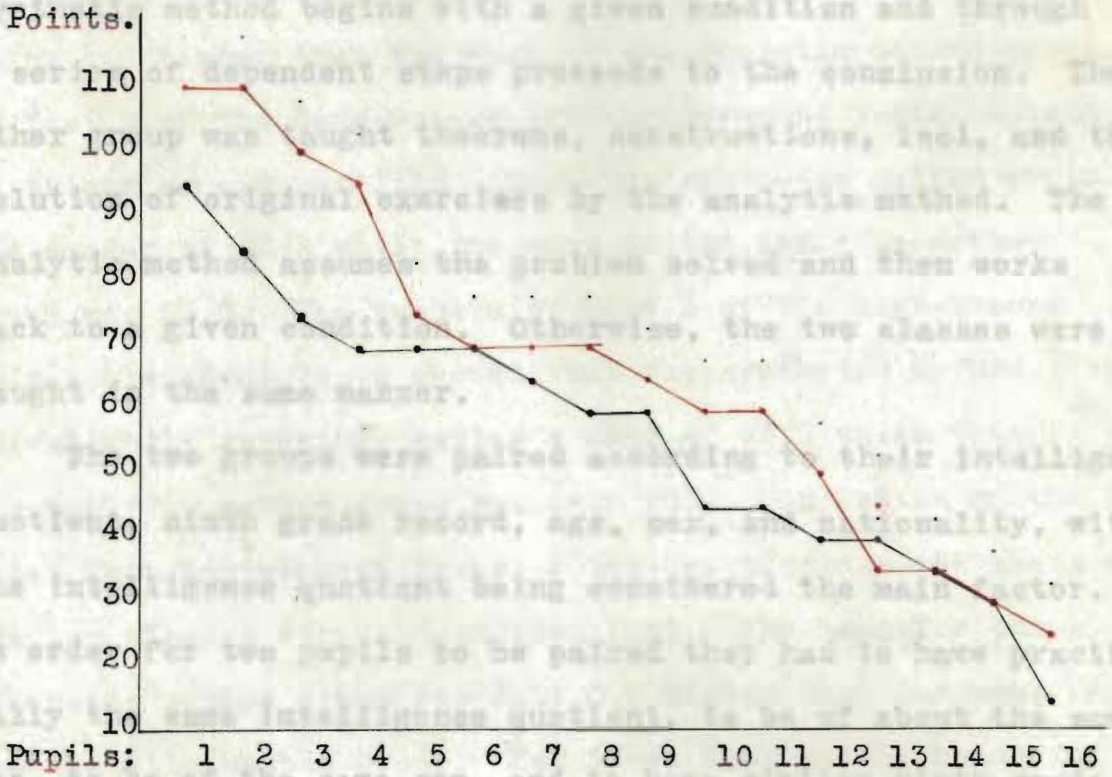
TABLE XIII... SHOWING RESULTS OF TEST XII...A TEST ON THE SOLUTION OF ORIGINAL EXERCISES.

	SYNTHETIC	ANALYTIC	Difference	
Pair	Score	Score	S	A
1	73	96		23
2	83	72	11	
3	69	101		32
4	66	108		42
5	33	77		44
6	70	69	1	
7	43	26	17	
8	72	51	21	
9	58	68		10
10	94	65	29	
11	60	112		52
12	31	30	1	
13	42	60		18
14	43	60		17
15	38	33	5	
16	16	33		17
Total points	121			
Range	16-94	26-112		
Mean	55.7	66.3		10.6
Mean upper 1/4 of class	80.5	104.3		23.8
Mean middle 1/2 of class	56.4	65.3		8.9
Mean lower 1/4 of class	29.5	30.5		1.0

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Score for pupil of pair one for the synthetic method was 73; for analytic method 96. The difference is 23 in favor of the analytic method. Read remainder of table accordingly.

GRAPH XII

SHOWING SCORES OF PUPILS ON TEST XII



Read table thus: Black indicates synthetic method group; red indicates analytic method group. The score of pupil 1 in the synthetic method group was 95, and the score of pupil 1 in the analytic method group 110. The scores are arranged in descending order.

CHAPTER IV

SUMMARY AND CONCLUSION

For this study the pupils were divided into two groups, one group was taught theorems, constructions, loci, and the solution of original exercises by the synthetic method. The synthetic method begins with a given condition and through a series of dependent steps proceeds to the conclusion. The other group was taught theorems, constructions, loci, and the solution of original exercises by the analytic method. The analytic method assumes the problem solved and then works back to a given condition. Otherwise, the two classes were taught in the same manner.

The two groups were paired according to their intelligence quotient, ninth grade record, age, sex, and nationality, with the intelligence quotient being considered the main factor. In order for two pupils to be paired they had to have practically the same intelligence quotient, to be of about the same age, to be of the same sex, and to have similar ninth grade records. In fact there was close agreement in all these phases. Due to the care used in pairing only sixteen pairs were obtained from a total of fifty-six students.

A test was given on the average of once every four or five weeks, depending upon the completion of a unit. Of the eight tests given, exclusive of the final test, five were the five tests of the Lane-Greene Unit-Achievement Tests, while

the remaining three were devised by the writer.

Let us direct our attention to the results of the Lane-Greene Unit-Achievement Tests. On the first test--a test on fundamental ideas of geometry--the results for the two classes were about equal--the mean for the synthetic method group being 1.2 higher than the mean of the analytic method group. On Test 2 of the Lane-Greene Unit-Achievement Tests--a test on triangles and parallel lines--the synthetic method group had a mean of 24.3 while the mean of the analytic method group was 23.0. The results of Test 3 of the Lane-Greene Unit-Achievement Tests showed that the synthetic method group was slightly superior, having a mean of 22.5 while that for the analytic method group was only 20.0. On Test 4 of the Lane-Green Unit-Achievement Tests--a test on circles--the means for the two classes were almost identical. The mean for the synthetic method group was only 0.3 higher than the mean for the analytic method group. Yet if we look at the means for the upper one fourth of the two groups, we find that the mean for the analytic method group is 2.0 higher than that for the synthetic method group; while the mean for the lower one fourth of the class is 3.0 lower than the mean of the corresponding one fourth of the synthetic method group. This seems to indicate that the more intelligent pupils have grasped the analytic method, but that the duller pupils have not yet grasped it. In the first three tests of the Lane-Greene Unit-Achievement Tests the results were quite uniform between

the two classes for the upper one fourth, middle one half, and lower one fourth of the classes.

Let us see what the results were for Test 5 of the Lane-Greene Unit-Achievement Tests--a test on similar polygons and proportion. The mean for the synthetic method group was 13.8 and the mean for the analytic method group was 16.3 which is 2.5 in favor of the analytic method group. The mean for the upper one fourth of the analytic method group was 3.5 higher than the mean for the corresponding one fourth of the synthetic method group. The mean for the lower one fourth of the analytic method group was 1.0 above the mean of the corresponding one fourth of the group using the synthetic method. The results of this test indicate that the analytic method group is now doing better work than the synthetic method group and especially in the upper one fourth of the classes. These tests seem to show that the duller pupil does not grasp the analytic method until late in the second semester.

The first test that the writer devised was given during the ninth week of school and consisted of questions on facts and definitions, two constructions, and one original exercise. The results showed that the analytic method group had a mean of 5.1 higher than that of the synthetic method group. Three students of the synthetic method group made rather poor scores as compared with the remainder of the class which probably accounts for the difference in the respective means of the two classes.

The second test that the writer devised was a test on original exercises. This was the fourth test given during the school year. The mean for the synthetic method group was 32.75 while that for the analytic method group was 36.0, a difference of 3.25 in favor of the analytic method group.

The third and last of the writer's tests was a test on facts, definitions, and constructions. The means for the two groups were almost identical. The mean for the synthetic method group was only 0.4 higher than the mean for the analytic method group.

Let us consider the results of the four parts of the final test, which were over the entire year's work and were given during the last week of school.

Test IX was a test on geometric facts, such as "Where do the medians of a triangle meet?" There were forty questions. The mean for the synthetic method group was 21.8, while the mean for the analytic method group was 19.4. There was no significant difference between the means of the upper one fourth, middle one half, and lower one fourth of the classes.

Test X, the second part of the final, was a test on definitions. The analytic method group had a mean of 36.9 and the synthetic method group a mean of 34.1. If we consider the results of both tests IX and X, there is practically no difference between the two groups. This result might be expected because the analytic and synthetic methods are not used in teaching definitions and facts, and could influence

the result only indirectly, and it is reasonable to believe that this influence would be negligible. These results seem to indicate that the pairings are reliable.

The third part of the final test, designated as Test XI in the tables, was a test on constructions and loci. The results of this test show that the mean for the synthetic method group was 41.3 and that for the analytic method group 43.65. The analytic method group would probably have had a higher mean if the test had been more difficult, as the entire upper one fourth of the analytic method group scored a perfect grade while only one pupil of the synthetic method group did as well.

Test XII, a test on the pupil's ability to solve original exercises, shows a very significant difference in favor of the analytic method group. Original exercises constitute a large portion of the work in plane geometry, and thus the method of teaching this material is important. The group that was taught by the analytic method had a mean of 66.5 while that of the synthetic method group was only 55.7. This shows that the results of the analytic method group were almost twenty per cent better than the results of the synthetic method group. But in the upper one fourth of the analytic method group the results were almost thirty per cent better than those of the upper one fourth of the synthetic method group. The mean for the lower one fourth of the analytic method group was only three and one-third per cent better than

TABLE XIV...SHOWING THE RESULTS OF THOSE TESTS WHICH INVOLVED LITTLE OR NO USE OF THE SYNTHETIC OR ANALYTIC METHOD.

	SYNTHETIC				ANALYTIC			
		Upper	Middle	Lower		Upper	Middle	Lower
Test	Mean	25%	50%	25%	Mean	25%	50%	25%
I	16.6	22.25	16.25	11.5	15.4	20.75	15.25	10.25
II	15.0	23.25	13.75	9.0	20.1	27.0	21.5	12.0
III	24.3	36.5	23.6	13.5	23.0	35.0	21.4	14.25
IV	22.5	33.75	22.9	11.5	20.0	28.5	19.9	11.75
VI	9.0	15.0	8.5	4.0	8.6	14.75	9.5	3.25
VII	18.5	24.25	18.75	12.25	18.2	26.25	18.6	9.25
VIII	13.8	21.25	13.6	6.25	16.3	25.0	16.5	7.25
IX	21.8	31.05	22.6	11.05	19.4	28.0	20.25	9.5
X	34.1	42.25	35.0	24.25	35.9	45.0	38.1	24.25
Mean:	19.51				19.63			
Mean:		27.75				27.78		
Mean:			19.44				20.11	
Mean:				11.47				11.31

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Each number indicates a mean .

TABLE XV...SHOWING RESULTS OF TESTS ON CONSTRUCTIONS, LOCI, AND ORIGINAL EXERCISES.

	SYNTHETIC				ANALYTIC			
		Upper	Middle	Lower		Upper	Middle	Lower
Test	Mean	25%	50%	25%	Mean	25%	50%	25%
IV	32.75	42.0	32.1	24.75	36.0	44.75	36.0	27.0
XI	41.31	46.89	42.18	33.93	43.65	48.0	45.18	36.18
XII	55.7	80.5	56.4	29.5	66.3	104.3	65.3	30.5
Mean:	43.25				48.65			
Mean:		56.46				65.68		
Mean:			43.56				48.83	
Mean:				29.39				31.23

Read table thus: Black indicates synthetic method; red indicates analytic method. Score is expressed in points. Each number indicates a mean.

the mean for the lower one fourth of the synthetic method group. This test seems to indicate that the analytic method for the solution of original exercises is much more efficient in the case of the average and above average student, and that there is little difference in the merits of the two methods in the case of the duller pupil.

Table XIV, on the following page, is a compilation of the results of all tests which involved little or no use of the analytic or synthetic method. This table includes all tests except those on constructions, loci, and the solution of original exercises. This table shows that the results for both groups were almost identical. The synthetic method group had a mean of 19.51 on these nine tests, while the mean for the analytic method group was 19.63, a difference of only 0.12. In the upper one fourth of the classes the mean for the analytic method group exceeded the mean for the synthetic method group by only 0.03, while for the lower one fourth, the mean of the analytic method group was 0.16 below that of the synthetic method group. These results again emphasize the care used in pairing, and thus enhances the validity of our conclusions on the respective merits of the analytic and synthetic methods.

Table XV, on the following page, is a compilation of tests IV, XI, and XII. These tests were on constructions, loci, and the solution of original exercises. It may be observed from this table that the results of the analytic method group were

much better than those of the synthetic method group. The mean for the synthetic method group was 43.25 and the mean for the analytic method group was 48.65 which is 12.5% greater. The mean for the upper one fourth of the analytic method group exceeded the mean for the corresponding one fourth of the synthetic method group by 16.3%; but in the lower one fourth of the classes the mean for the analytic method groups was only 6.3% better than the mean for the synthetic method group. This seems to indicate that the analytic method is superior to the synthetic method for solving original exercises, in making constructions, and finding loci.

CONCLUSIONS:

The data obtained indicates that:

1. The analytic method is better than the synthetic method for teaching constructions, loci, and original exercises.

2. The analytic method benefits the intelligent pupil more than the dull pupil.

3. The analytic method will not show better results than the synthetic method until the second semester.

4. I believe that it is reasonable to assume that the results would have favored the analytic method more if the writer had had the experience in teaching it which he has had teaching the synthetic method.

5. The greatest benefit to be derived from the use of the synthetic method is in the solution of original exercises.

APPENDIX

COPIES OF TESTS I - XII

LANE-GREENE UNIT-ACHIEVEMENT TESTS IN PLANE GEOMETRY

BY RUTH O. LANE AND H. A. GREENE

Score _____

Test 1. Fundamental Ideas of Geometry. *Form A*

Name _____ Grade_____ Date _____

Name of School_____ City_____ State_____

Directions to Student. This is a test to find out how well you have learned certain important facts of plane geometry.

The test consists of 34 exercises to be worked. In doing these exercises you will need a sheet of paper and a protractor. Your answers are to be brief and are to be written on the lines at the right of each exercise. Wherever possible write only the letters which stand for an angle, a figure, or a group of words.

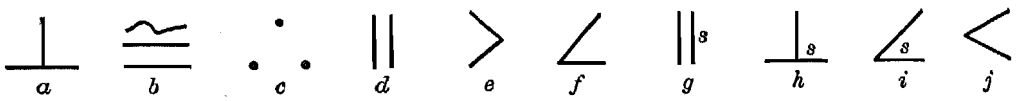
The following samples are answered correctly. Do the remaining exercises in the same manner. You will be given 35 minutes for this test.

SAMPLES — Answer

A. The symbol for *equals* is — =

B. The word *line* is used in geometry to mean: (a) a short line; (b) a long line; (c) a line of a given length; (d) a line of indefinite length; (e) a boundary line. — *d*

1. The symbol for *perpendicular* is the one marked by the letter _____



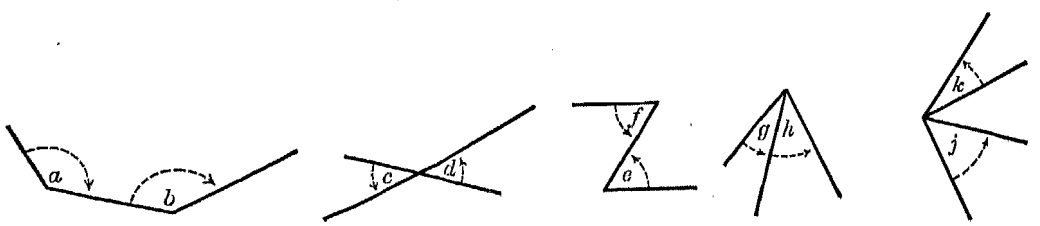
2. What name is given to the point where the sides of an angle meet? _____

3. "If equals are multiplied by equals the products are equal." This statement is called an _____

4. If a curved line and a straight line are drawn between two points, the shorter distance is represented by the (*two words*) _____

5. A geometric figure may be moved about in space without changing its size or _____

6. Which of these angles are vertical? _____

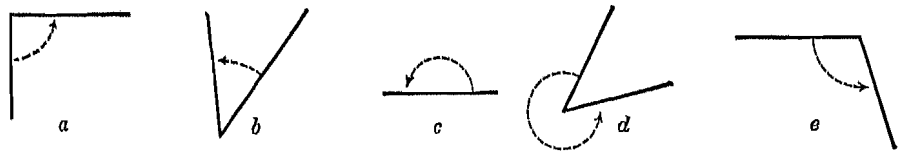


7. The number of straight lines which may be drawn connecting two points is: (a) none; (b) one; (c) two; (d) three; (e) an indefinite number. _____

Turn to page 2 and go right on working.

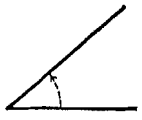
8. The complement of an angle of 35° is an angle of how many degrees? _____

9. Which of these angles is obtuse? _____



10. The distance from the center of a circle to the circle is called the _____

11. How many degrees are there in this angle? (*Use a protractor.*) _____

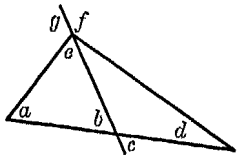


12. The word which may mean either a *theorem* or a *problem* is _____

13. The boundary lines of a square are line _____

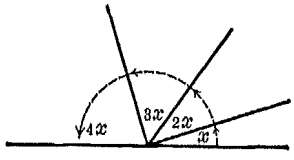
14. The whole is equal to the sum of all its _____

15. Name a pair of adjacent angles in this figure. _____

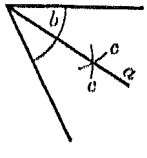


16. If a block of wood is moved from any position to another, the space it formerly occupied is called a geometric _____

17. How many degrees are there in angle x? (*You should not use the protractor here.*) _____



18. This angle has been bisected. Write the letters a, b, c so that they will show the order in which the lines they represent were drawn. _____

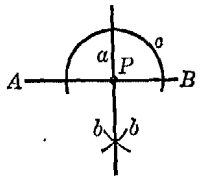


Turn to page 3 and go right on working.

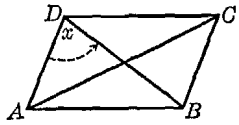
19. At 10 : 25 the minute hand of a clock has turned (5°, 25°, 150°, 180°, 210°) since 10 o'clock. ----------

20. A perpendicular to the line AB has been constructed at the point P. Write the letters a, b, c so that they will show the order in which the lines they represent were drawn. ----------

(If two lines are marked with the same letter, this means that the order in which the lines were drawn is interchangeable.)



21. Name angle x in one other way, using three letters already on the figure. ----------

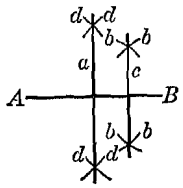


22. How many degrees are there in this angle? (*Use a protractor.*) ----------

23. What is formed by the intersection of two lines in a plane? ----------

24. If two adjacent supplementary angles are bisected, what kind of angle is formed by the bisectors? ----------

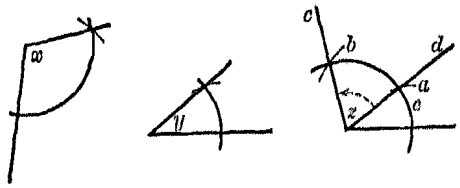
25. One fourth of the line AB has been found. Write the letters a, b, c, d to show the order in which the lines they represent were drawn. ----------



26. This () shows ----------

but this (————) shows ----------

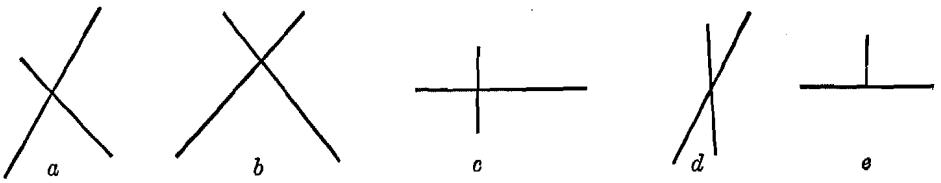
27. Angle z has been constructed equal to the difference between angle x and angle y. The radius used for arc e is the same as that for the arcs on angle x and angle y. Write the letters a, b, c, d, e so that they will show the order in which the lines they represent were drawn. ----------



Turn to page 4 and go right on working.

28. How many geometric surfaces has a chalk box? ----------

29. Which figure represents two lines bisecting each other? ----------



30. The word *geometry* means, in the Greek, the measurement of the ----------

31. How many degrees are in an angle which is formed by $1\frac{1}{2}$ rotations of a line in a plane about a fixed point? ----------

32. What is the size of an angle which is nine times the size of its complement? (An equation should be used in obtaining the answer.) ----------

33. A point in a geometric figure is indicated by what kind of letter? ----------

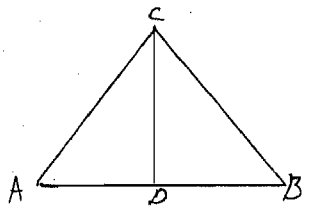
34. Two right angles have a common vertex, and a side of one of the right angles lies between the sides of the other. This forms three angles. Two pairs of the angles thus formed are complementary, and the other pair are ----------

END OF THE TEST; CLOSE YOUR PAPER.

Score = Number of exercises correct = ----------

(Total possible score = 34 points)

TEST II

1. Give the three steps in a formal proof.
2. How many points are necessary to fix the position of a straight line?
3. Draw two adjacent angles.
4. What are perpendicular lines?
5. 

   Given: ac=cb
   cd bisects ∠C

   Prove: △ADC ≅ BDC

6. List the three conditions that are necessary to prove two triangles congruent.
7. Construct an angle equal to a given angle.
8. Bisect an angle.
9. What pairs of angles are equal when two parallel lines are cut by a transversal?
10. Through a given point construct a line parallel to a given line.
11. What is the most common method of proving two line segments or two angles equal?
12. What is a hypotenuse?
13. What are supplementary angles?

Value assigned to each exercise:

1--- 3 points	6--- 6 points	10--- 3 points
2--- 1 point	7--- 3 points	11--- 2 points
3--- 2 points	8--- 2 points	12--- 1 point
4--- 2 points	9--- 2 points	13--- 1 point
5--- 2 points	Total points: 30	

LANE-GREENE UNIT-ACHIEVEMENT TESTS IN PLANE GEOMETRY

BY RUTH O. LANE AND H. A. GREENE

Score	
Part 1	____
Part 2	____
Part 3	____
Part 4	____
Total	____

Test 2. Parallel Lines and Triangles. *Form A*

Name _____ Grade _____ Date _____

Name of School _____ City _____ State _____

General Directions. This is a test to find out how well you can work certain problems and theorems dealing with parallel lines and triangles. Some of these theorems may be entirely new to you, but do not let this make any difference. You are not required to prove the theorems.

This test is in four parts. *Part 1* is a test of your ability to read a theorem and to state *what is to be proved*. *Part 2* calls upon you to read a theorem and to state *what is given* in it. *Part 3* is a test of your ability to *prove theorems*. *Part 4* is a series of *miscellaneous exercises* about parallel lines and triangles.

Directions are given for each part of the test.

Directions to Student for Part 1. This part of the test consists of a series of ten statements of theorems and problems. Words or phrases in each statement are grouped together by means of brackets which are numbered. *You are to read each exercise carefully and write on the dotted line at the right the number of the bracketed portion of the exercise which contains the statement of what is to be proved in the theorem or problem.* It may be necessary to write more than one number in answering some of the exercises. The sample exercise is answered correctly. Answer the remaining exercises in a similar manner. You will be given 4 minutes for this part of the test.

SAMPLE

Two triangles (1) are congruent (2) if three sides of one triangle are equal respectively (3) to three sides of the other. (4)

Answer 1, 2

Part 1. Indicating what is to be Proved

1. If (1) two right triangles have the hypotenuse and the acute angle of one (1) equal respectively to the hypotenuse (2) and an acute angle of the other (3), the triangles are congruent (4). _____

2. Two triangles (1) are congruent (1) if (2) they have two angles and the included side of one (2) equal respectively (3) to two angles and the included side of the other (4). _____

3. If (1) two parallel lines are cut (1) by a transversal (2), the alternate interior angles (3) are equal and the alternate exterior angles (4) are equal. _____

4. In an isosceles triangle (1) the angles (2) opposite the equal sides (3) are equal (4). _____

5. If (1) two lines in the same plane are cut (1) by a transversal (2) so that the corresponding angles (3) are equal, the lines (4) are parallel. _____

6. If (1) a given four-sided figure has its vertices on the circumference of a circle (2), its opposite angles (3) are supplementary (4). _____

7. A diagonal (1) of a parallelogram bisects two angles (1) and is at right angles (2) to the other diagonal and is equal to the other diagonal (3) if the parallelogram is a square (4). _____

8. When the median (1) of a right triangle is drawn from the vertex (2) of a right angle (3), it is one half of the hypotenuse (4). _____

9. When a line is drawn parallel to the base (1) of an isosceles triangle cutting the equal sides (2), it forms with these sides (3) another isosceles triangle (4). _____

10. If an angle (1) of one triangle is equal to an angle (2) of another, the triangles (3) are to each other as the products of the sides (4) forming the equal angles. _____

Score on Part 1 = Number right = _____

Do not work on Part 2 until told to do so.

Directions to Student for Part 2. This part of the test is quite similar to Part 1. *You are to read each exercise carefully and write on the dotted line at the right the number of the bracketed portion of the exercise which contains the statement of what is given in the theorem or problem.* It may be necessary to write more than one number in answering some of the exercises. The sample exercise below is answered correctly. Answer the remaining exercises in this part in a similar manner. You will be given 4 minutes for this part.

SAMPLE

Two triangles are congruent (1) if three sides of one triangle (2) are equal respectively (3) to three sides of the other. (4) — Answer: 2, 3, 4

Part 2. Indicating what is Given

1. If (1) the hypotenuse and a side of a right triangle (1) are respectively equal (2) to the hypotenuse and a side of another, (3) the triangles are congruent. (4) _____

2. A line is parallel (1) to the third side of a triangle (2) if it cuts the other two sides (3) of the triangle (4) proportionately. _____

3. If two lines in the same plane (1) are cut by a transversal (2) so that the alternate interior angles (3) are equal, the lines are parallel. (4) _____

4. Two lines in the same plane cut by a transversal (1) are parallel (2) if the corresponding angles (3) are equal. (4) _____

5. The bisector of the exterior angle (1) at the vertex (2) of an isosceles triangle is parallel (3) to the base of the triangle. (4) _____

6. The bisector of an interior angle (1) of a triangle divides the opposite side (2) into segments which are proportional (3) to the adjacent sides. (4) _____

7. The point (1) in which the bisectors (2) of the angles of an equilateral triangle meet is equidistant (3) from the sides of the triangle. (4) _____

8. The medians (1) of a triangle coincide with the altitudes, coincide (2) with the bisectors of the angles, and coincide with the perpendicular bisectors (3) of the sides of the triangle, if the triangle is (4) equilateral. _____

9. The sum (1) of the number of degrees in the interior angles of a triangle (2) is equal (3) to 180°. (4) _____

10. The areas (1) of two triangles are to each other as the squares (2) of any two corresponding sides if the triangles (3) are similar. (4) _____

Score on Part 2 = Number right = _____

Do not work on Part 3 until told to do so.

Directions to Student for Part 4. The sample given below is answered correctly. Answer the other exercises in a similar manner. You will be given 12 minutes for this part.

SAMPLE

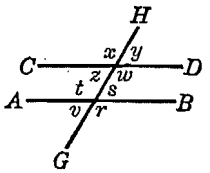
	Answer
The side of a triangle upon which it is considered to rest is called its	*base*

Part 4. Miscellaneous Exercises

1. Lines in the same plane which are always the same distance apart are said to be _____

2. If two angles of a triangle are each 35°, how large is the third angle? _____

3. In this figure what angle with z forms a pair of alternate interior angles? _____



4. In the figure above, what angle with v forms a pair of alternate exterior angles? _____

5. If, in the figure above, angle z is 55°, how large is angle r? _____

6. What angle with r forms a pair of corresponding angles in this drawing of two parallel lines cut by a transversal? (*See figure for Exercise 3.*) _____

7. In a right triangle the side opposite the right angle is called the _____

8. In a certain isosceles triangle the vertex angle is 20°. How large are the other angles? _____

9. Through a given point not on a straight line, how many lines can be drawn perpendicular to the given line? _____

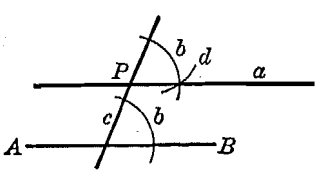
10. How many degrees in each angle of an equilateral triangle? _____

11. An acute triangle has (a) one angle larger than 90°; (b) only one angle less than 90°; (c) only two angles less than 90°; (d) two angles more than 90°; (e) three angles less than 90°. (Answer *a*, *b*, *c*, *d*, or *e*.) _____

12. Dotted lines on a figure used to assist in proving a theorem are called _____

13. A line has been constructed parallel to AB through the point P. Write the letters *a*, *b*, *c*, *d* so that they will show the order in which the lines they represent were drawn. _____

(When the same letter is used to mark two lines, it means that these two constructions are interchangeable in order.)



14. Two lines in the same plane perpendicular to the same line are _____

15. A line perpendicular to one of two lines which are perpendicular to each other and in the same plane is related in what way to the other? _____

Turn to page 6 and go right on working.

In answering exercises 6 to 11 inclusive state which of the following tests for congruency you would use in the proof of the theorems or the solution of the exercises. Use the letters *a*, *b*, *c*, or *d*. You may need to make sketches of the figures.

Two triangles are congruent if they have the following parts of each respectively equal:

a. Two sides and the included angle
b. Two angles and any side
c. Three sides
d. Hypotenuse and any other side

Tests for Congruency of Triangles

6. Any point on the perpendicular bisector of a line is equidistant from the ends of the line. (Answer *a*, *b*, *c*, or *d*.) _____

7. If the four interior angles formed by two parallel lines and a transversal are bisected, the triangles thus formed are congruent. _____

8. The bisectors of the equal angles of an isosceles triangle drawn to the opposite sides are equal. _____

9. If a figure of four sides has opposite sides equal, a line between any two opposite vertices divides the figure into two congruent triangles. _____

10. Any point equidistant from the sides of an angle determines with the vertex the bisector of the angle. _____

11. Which of the above tests for congruency is applied in finding the distance across a river by the use of congruent triangles laid out on the ground? _____

Recognition of the Converse of a Theorem

12. If the alternate interior angles formed by two lines and a transversal are equal, the lines are parallel. *Which of the following statements is the converse of this theorem?* _____

a. If the alternate interior angles formed by two lines and a transversal are not equal, the lines are not parallel.
b. The alternate interior angles formed by two lines and a transversal are equal if the lines are parallel.
c. If the alternate interior angles formed by two lines and a transversal are equal, the lines are not parallel.
d. Two lines cut by a transversal are parallel if the alternate interior angles are equal.
e. If the alternate exterior angles formed by two lines and a transversal are equal, the lines are parallel.

13. The perpendicular drawn from the vertex of a right triangle to the hypotenuse divides the triangle into two similar triangles. *Which of the following statements is the converse of this theorem?* _____

a. A right triangle is divided into two similar triangles by a perpendicular drawn from the vertex of the right angle to the hypotenuse.
b. The perpendicular drawn from the vertex of a right triangle to the hypotenuse does not divide the triangle into two similar triangles.
c. If a line divides a right triangle into two similar triangles, it must be a perpendicular drawn from the vertex of the right angle upon the hypotenuse.
d. If a perpendicular is not drawn from the vertex of a right triangle perpendicular to the base, it does not divide the triangle into two similar triangles.
e. A perpendicular drawn from the vertex of any angle of a right triangle divides the triangle into two similar triangles.

Score on Part 3 = Number right = _____

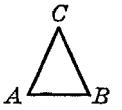
Do not work on Part 4 until told to do so.

Directions to Student for Part 3. In this part of the test you are asked to do a number of different things. In the first three exercises you are to state in specific terms *what is to be proved in the theorem.* The sample below is answered correctly. Later you are asked (1) *to find errors in the statements of a proof,* (2) *to apply the tests for congruency of triangles,* and (3) *to recognize converse statements of theorems.*

Additional directions are given where necessary. You will be given 18 minutes for this part.

SAMPLE **Answer**

Two angles of a triangle are equal if two sides are equal. *State what is to be proved in terms of this figure.*

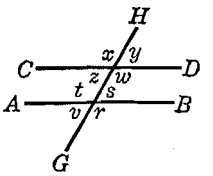


$\angle A = \angle B$

Part 3. Proving Theorems

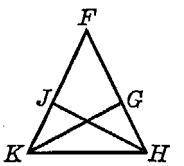
Stating what is to be Proved

1. If two lines in the same plane are cut by a transversal so that the corresponding angles are equal, the lines are parallel. *State what is to be proved in terms of this figure.*



2. If two parallel lines are cut by a transversal the two exterior angles on the same side of the transversal are supplementary. *State what is to be proved in terms of the figure above.*

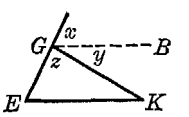
3. Perpendiculars drawn from the vertices of the equal angles of an isosceles triangle to the opposite sides are equal. *State what is to be proved in terms of this figure.*



Finding the Error in Statement of Proof

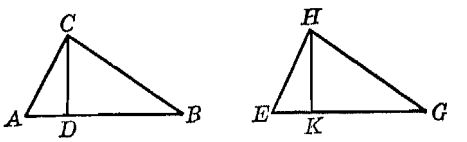
4. The sum of the angles of a triangle is 180°. (GB is drawn parallel to EK.) *Which of the following statements in the proof of this theorem is wrong, useless, or out of order?* (*The reasons are omitted.*)

a. $\angle x = \angle E$
b. $\angle x = \angle y$
c. $\angle y = \angle K$
d. $\angle x + \angle y + \angle z = 180°$
e. $\angle z + \angle K + \angle E = 180°$



5. Two triangles are congruent (equal in every respect) if a side, an adjacent angle, and an altitude upon that side in one are equal respectively to the corresponding parts in the other. *Which statement in the proof of this theorem is wrong, useless, or out of order?*

a. $\angle A = \angle E$
b. $\angle ADC = \angle EKH$
c. $CD = HK$
d. $\triangle ADC \cong \triangle EKH$
e. $CB = HG$
f. $AC = EH$
g. $AB = EG$
h. $\triangle ABC \cong \triangle EGH$



Turn to page 4 and go right on working.

16. The parts of two congruent figures which occupy the same relative position are described as _____

17. The bisectors of the angles opposite the equal sides of a right isosceles triangle form an angle of (60°, 90°, 135°, 150°, varying size). _____

18. If two angles have their sides parallel right side to right side and left side to left side the angles are _____

19. If two sides of a triangle are 5 inches and 20 inches, the third side cannot be (15, 18, 20, 22, 24) inches. _____

20. How many altitudes has a triangle? _____

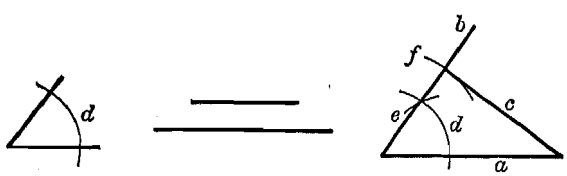
21. In order to construct one triangle equal to another, one need measure only three _____

22. The placing of one figure upon another in the proof of a theorem is called _____

23. If two angles have their sides respectively perpendicular right side to left side and left side to right side, the angles are _____

24. Two angles of a triangle are 20° and 80°. How large is the opposite exterior angle? _____

25. A triangle has been constructed when two sides and the included angle were given. Write the letters *a*, *b*, *c*, *d*, *e*, *f* so that they will show the order in which the lines they represent were drawn. _____



26. A median of any triangle (a) is perpendicular to a side of the triangle; (b) is the distance from the vertex to the middle of the opposite side; (c) bisects an angle; (d) joins the midpoint of two sides; (e) is the perpendicular bisector of a side. (Answer *a*, *b*, *c*, *d*, or *e*.) _____

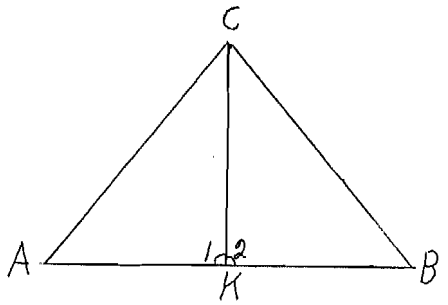
27. If one angle of a right triangle is 60°, then two of the sides of the triangle are in the ratio (a) 1 to 1; (b) 1 to 2; (c) 1 to 3; (d) 1 to 4; (e) no fixed ratio. _____

END OF THE TEST; CLOSE YOUR PAPER.

Score on Part 4 = Number right = _____

TEST IV

1



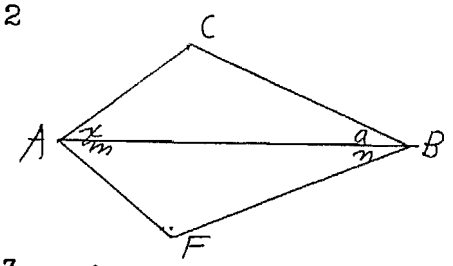
Given: KC bisects AB

$KC \perp AB$

Prove: $AC = BC$

Proof:

2



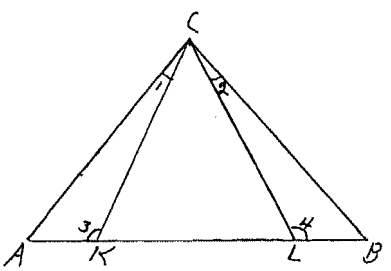
Given: $x = m$

$a = n$

Prove: $AC = AF$

Proof:

3



Given: $AC = BC$

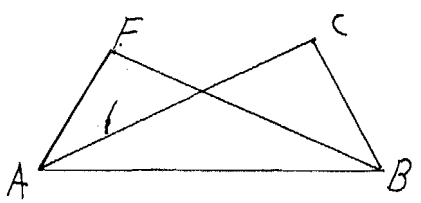
$\angle 1 = \angle 2$

Prove: $\angle 3 = \angle 4$

$CK = CL$

Proof:

4



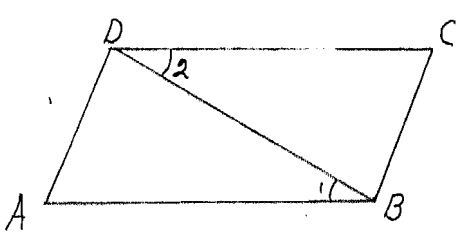
Given: $AF = BC$

$AC = BF$

Prove: $\angle F = \angle C$

Proof:

5



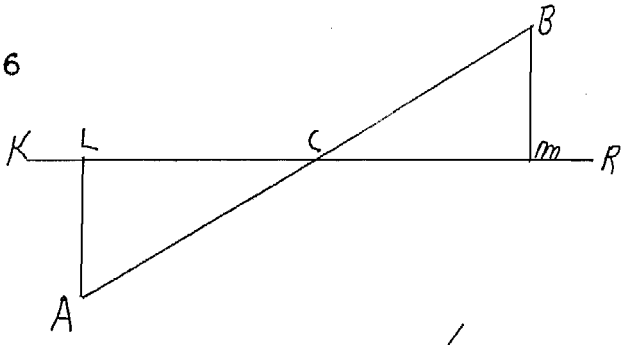
Given: $AB = DC$

$AD = BC$

Prove: $AB \parallel DC$

Proof:

6



Given: C midpoint AB

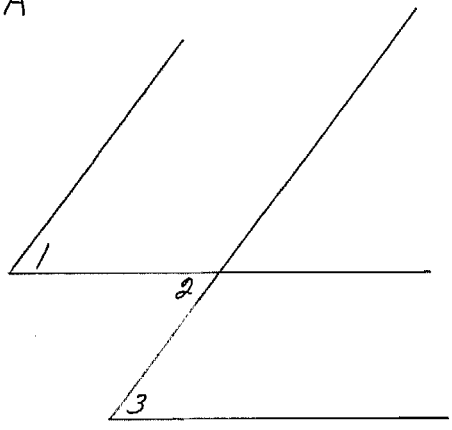
BM ⊥ KR

AL ⊥ KR

Prove: ∠A ≅ ∠B

Proof:

7



Given: AB // ED

AC // EF

Prove: ∠1 ≅ ∠3

Proof:

Value assigned to each problem:-

1 7½ points

2 7½ points

3 9 points

4 7½ points

5 9 points

6 6 points

7 4½ points

The point value of a problem depemds upon the number of statements required to solve the problem.

LANE-GREENE UNIT-ACHIEVEMENT TESTS IN PLANE GEOMETRY

BY RUTH O. LANE AND H. A. GREENE

Score	
Part 1	____
Part 2	____
Part 3	____
Part 4	____
Total	____

Test 3. Rectilinear Figures. *Form A*

Name _____ Grade_____ Date_____

Name of School _____ City_____ State_____

General Directions. This is a test to find out how well you can work certain problems and theorems dealing with rectilinear figures. Some of these theorems may be entirely new to you, but do not let this make any difference. This test is in four parts. Directions are given for each part of the test.

Directions to Student for Part 1. This part of the test consists of a series of ten statements of theorems. Words or phrases in each statement are grouped together by means of brackets which are numbered. *You are to read each exercise carefully and write on the dotted line at the right the number of the bracketed portion of the exercise which contains the statement of what is to be proved in the theorem.* It may be necessary to write more than one number in answering some of the exercises. The sample exercise below is answered correctly. Answer the remaining exercises in this part in a similar manner. You will be given 3 minutes for this part of the test.

SAMPLE — **Answer**

If (1) two lines in the same plane are cut by a transversal (2) so that the alternate interior angles are equal, (3) the lines are parallel. (4) — *3, 4*

Part 1. Indicating what is to be proved

1. If (1) two triangles have two angles of one (2) equal respectively to two angles of the other triangle, (3) the third angles of the triangles (4) are equal. _____

2. If (1) from any point within an equilateral triangle (2) perpendiculars are drawn to the sides, and if (3) an altitude of the triangle is drawn, (4) the altitude equals the sum of the three perpendiculars. _____

3. (1) Two parallelograms are congruent if (2) two sides and the included angle of one (3) equal respectively (4) two sides and the included angle of the other. _____

4. If (1) lines are drawn from any point within a triangle (2) to the extremities of one side, (3) these lines form an angle (4) greater than the angle formed by the other two sides of the triangle. _____

5. If (1) the vertices of one parallelogram (2) lie upon the four sides of another, (3) the parallelograms (4) have the same center. _____

6. (1) A trapezoid is isosceles if (2) the diagonals (3) of the trapezoid (4) are equal. _____

7. If (1) a parallelogram is not a rectangle, (2) the diagonals (3) of the parallelogram (4) are unequal. _____

8. (1) The diagonals of a parallelogram do not form (2) four congruent triangles if (3) the parallelogram (4) does not have equal sides. _____

9. If (1) a triangle is isosceles, (2) the angles opposite (3) the equal sides (4) are equal. _____

10. If (1) a line segment is parallel to the bases of a trapezoid (2) and bisects one of the non-parallel sides, (3) the line bisects the other side also (4) and is equal to one half the sum of the bases. _____

Score on Part 1 = Number right = _____

Do not work on Part 2 until told to do so.

Directions to Student for Part 2. This part of the test is quite similar to Part 1. *You are to read each exercise carefully and write on the dotted line at the right the number of the bracketed portion of the exercise which contains the statement of what is given in the theorem.* It may be necessary to write more than one number in answering some of the exercises. The sample exercise below is answered correctly. Answer the remaining exercises in this part in a similar manner. You will be given 3 minutes for this part of the test.

SAMPLE **Answer**

If [two lines in the same plane are cut by a transversal]$_1$ [so that the alternate interior angles are equal]$_2$, [the lines]$_3$ [are parallel.]$_4$ 1, 2

Part 2. Indicating what is Given

1. If [through any point]$_1$ [in the bisector of an angle]$_2$ [lines are drawn parallel to the sides of the angle]$_3$, [these parallel lines form with the sides of the angle a rhombus.]$_4$ _____

2. [The figure is a parallelogram]$_1$ [or a trapezoid]$_2$ if [a quadrilateral has two adjacent angles which are supplementary.]$_3$ _____

3. If [two lines]$_1$ [are not parallel]$_2$ and [are cut by a transversal,]$_3$ [the alternate interior angles are not equal.]$_4$ _____

4. [An isosceles trapezoid]$_1$ [has opposite angles supplementary.]$_2$ _____

5. [The bisectors of two adjacent angles]$_1$ [of a parallelogram]$_2$ [are perpendicular]$_3$ [to each other.]$_4$ _____

6. When [an isosceles triangle has a line bisecting a base angle]$_1$ and [one half the base angle is equal to the vertex angle,]$_2$ [the bisector of either base angle divides the triangle]$_3$ [into two isosceles triangles.]$_4$ _____

7. [The line segments joining the mid-points]$_1$ [of the opposite sides]$_2$ [of a quadrilateral]$_3$ [bisect each other.]$_4$ _____

8. [The perpendiculars from two vertices of a triangle]$_1$ [upon the median]$_2$ [drawn from the third vertex]$_3$ [are equal.]$_4$ _____

9. [A parallelogram is formed]$_1$ if [from any point in the base of an isosceles triangle]$_2$ [lines parallel to the equal sides]$_3$ [are drawn.]$_4$ _____

10. [A square and its diagonals]$_1$ [form four isosceles triangles,]$_2$ [eight angles of forty-five degrees,]$_3$ and [four angles of ninety degrees.]$_4$ _____

Score on Part 2 = Number right = _____

Do not work on Part 3 until told to do so.

Directions to Student for Part 4. The sample given below is answered correctly. Answer the other exercises in a similar manner. You will be given 12 minutes for this part of the test.

SAMPLE

The side of a triangle upon which it is considered to rest is called its — **Answer** *base*

Part 4. Miscellaneous Exercises

1. If one angle of a parallelogram is 40°, how large is the opposite angle? ---

2. How many sides has an octagon? ---

3. If $b < c < d$, what conclusion can you draw regarding b and d? ---

4. What is the number of diagonals which can be drawn in a polygon of seven sides from any one vertex? ---

5. The formula for the number of degrees in each angle of a polygon when all angles are equal is: (a) $\frac{(n-2)180}{n}$; (b) $180n - 180$; (c) $\frac{180n - 180}{n}$; (d) $(n-2)2$ rt. $\angle s$; (e) $\frac{180n}{n-2}$. (Answer *a*, *b*, *c*, *d*, or *e*.) ---

6. The angles of an equiangular polygon of six sides are each (90°, 108°, 112°, 120°, 135°). ---

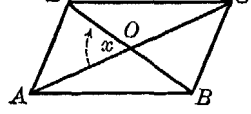
7. A quadrilateral is a parallelogram but not necessarily a rhombus or rectangle if: (a) its diagonals are equal; (b) its angles are right angles; (c) its diagonals bisect the angles; (d) its diagonals are perpendicular to each other; (e) its diagonals bisect each other. ---

8. By proof we know that both pairs of opposite sides of a parallelogram are ---

9. Two sides of one triangle are equal respectively to two sides of another triangle, but the included angle of the first is greater than the included angle of the second. How does the third side of the first compare with the third side of the second? ---

10. If the side JK is less than the side HJ of $\triangle HJK$ it follows that: (a) $\angle J > \angle K$; (b) $\angle H > \angle K$; (c) $\angle K > \angle J$; (d) $\angle H < \angle K$; (e) $\angle H > \angle J$. (Answer *a*, *b*, *c*, *d*, or *e*.) ---

11. If in the $\square ABCD$ $\angle ACD$ is 30° and $\angle ABO$ is 25°, how many degrees does $\angle x$ contain? ---



12. How many degrees are there in the sum of the angles of a polygon of seven sides? ---

13. What is the total number of diagonals which can be drawn in a pentagon? ---

Turn to page 6 and go right on working.

In answering exercises 5 to 13 inclusive state which of the following tests for congruency you would use in the proof of the theorems stated. Use the letters *a*, *b*, *c*, or *d*. You may need to make sketches of the figures.

Two triangles are congruent if they have the following parts respectively equal:

a. Two sides and the included angle.
b. Two angles and any side.
c. Three sides.
d. Hypotenuse and any other side.

Tests for Congruency of Triangles

5. If the diagonals of a quadrilateral bisect each other, the figure is a parallelogram. (Answer *a*, *b*, *c*, or *d*.) _____

6. The opposite sides of a parallelogram are equal. _____

7. The diagonals of a rhombus bisect the angles whose vertices they join. _____

8. Any point on the bisector of an angle is equidistant from the sides of the angle. _____

9. The altitudes upon a diagonal of a parallelogram from two opposite vertices are equal. _____

10. The diagonals of an isosceles trapezoid are equal. _____

11. If two altitudes of a triangle are equal, the triangle is isosceles. _____

12. Two isosceles triangles are congruent if they have their bases and one of the adjoining angles equal respectively. _____

13. If two parallelograms have one pair of opposite sides equal and the altitude between the other pair of opposite sides equal, the parallelograms are mutually equiangular. _____

Recognition of the Converse of a Theorem

14. If either diagonal of a parallelogram bisects one of the angles, the figure is equilateral. *Which of the following statements is the converse of this theorem?* (Answer *a*, *b*, *c*, *d*, or *e*.) _____

a. The parallelogram is equilateral if either diagonal bisects one of the angles.
b. If either diagonal of a parallelogram does not bisect one of the angles, the figure is not equilateral.
c. If either angle of a parallelogram is bisected by one of the diagonals, the figure is equilateral.
d. If a parallelogram is equilateral, either diagonal of the parallelogram bisects one of the angles.
e. If the parallelogram is equilateral, neither diagonal of the parallelogram bisects one of the angles.

15. If the pairs of opposite sides of a hexagon are equal and parallel, then the lines joining pairs of opposite vertices will pass through a common point. *Which of the following statements is the converse of this theorem?* _____

a. If the lines joining pairs of opposite vertices of a hexagon pass through a common point, the opposite sides of the hexagon are equal and parallel.
b. If the pairs of opposite sides of a hexagon are not equal and parallel, then the lines joining pairs of opposite vertices will not pass through a common point.
c. The lines joining pairs of opposite vertices of a hexagon will pass through a common point if the opposite sides of the hexagon are equal and parallel.
d. If the pairs of opposite sides of a hexagon are equal and parallel, then the lines joining pairs of opposite vertices will not pass through a common vertex.
e. If the pairs of opposite angles of a hexagon are equal and their sides parallel, then the lines joining pairs of opposite vertices will pass through a common point.

Score on Part 3 = Number right = _____

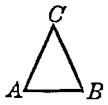
Do not work on Part 4 until told to do so.

Directions to Student for Part 3. In this part of the test you are asked to do a number of different things. In the first two exercises you are to state in specific terms *what is to be proved in the theorem.* The sample below is answered correctly. Later you are asked (1) *to find errors in the statements of a proof,* (2) *to apply the tests for congruency of triangles,* and (3) *to recognize converse statements of theorems.*

Additional directions are given where necessary. You will be given 20 minutes for this part of the test.

SAMPLE **Answer**

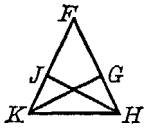
Two angles of a triangle are equal if two sides are equal. *State what is to be proved in terms of this figure.* $A = B$



Part 3. Proving Theorems

Stating what is to be Proved

1. If perpendiculars are drawn from the vertices of the equal angles of an isosceles triangle to the opposite sides, the two triangles thus formed which include the vertex angle are congruent. *State what is to be proved in terms of this figure.*

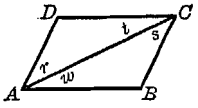


2. If the two opposite pairs of sides of a quadrilateral are equal, the opposite angles are equal. *State what is to be proved in terms of the figure for Exercise 3 below.*

Finding the Error in the Statement of Proof

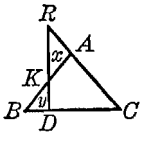
3. In the accompanying figure AB is given equal to DC and $\angle w = \angle t$. $ABCD$ has been proved below to be a parallelogram. *Which statement in the following proof is wrong, useless, or out of order?* (*The reasons are omitted.*)

a. $AB = DC$
b. $\angle w = \angle t$
c. $AB \parallel CD$
d. $\angle r = \angle s$
e. $ABCD$ is a parallelogram



4. In the accompanying figure $\triangle ABC$ is isosceles, K is the mid-point of AB, and KD is drawn perpendicular to BC. CA and DK are extended to meet at R. Below it is proved that $\angle R$ equals $\angle x$. *Which statement in the proof is wrong, useless, or out of order?*

a. $\angle BDK = 90°$
b. $\angle B + \angle y = 90°$
c. $\angle B + \angle x = 90°$
d. $2\angle B + 2\angle x = 180°$
e. $2\angle B + \angle CAB = 180°$
f. $4\angle B + 2\angle x + \angle CAB = 360°$
g. $2\angle x = \angle CAB$
h. $\angle x + \angle R = \angle CAB$
i. $\angle R = \angle x$



Turn to page 4 and go right on working.

14. A line perpendicular to a vertical plane is called a _____

15. If unequals are subtracted from equals, the results are unequal in the (*two words*) _____

16. An equilateral rectangle is a _____

17. Oblique lines are not _____

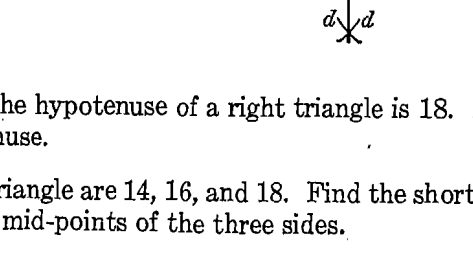
18. How many degrees in the sum of the exterior angles (formed by extending each side in succession through the vertex) of a polygon of seven sides? _____

19. How many degrees in each exterior angle of a polygon of ten sides? _____

20. A right triangle has been constructed using the given line e for the hypotenuse and the given acute $\angle B$. _____

Write the letters a, b, c, d, e, f, g, so that they will show the order in which the lines they represent were drawn.

(When the same letter is used to mark two lines it means that these two constructions are interchangeable in order.)



21. The length of the hypotenuse of a right triangle is 18. Find the length of the median to the hypotenuse. _____

22. The sides of a triangle are 14, 16, and 18. Find the shortest side of the triangle formed by joining the mid-points of the three sides. _____

23. The bases of a trapezoid are 8 and 18. Find the length of the line joining the mid-points of the diagonals. _____

24. One angle of a rhombus is 120° and the sides are each 6. What is the length of the shorter diagonal? _____

END OF THE TEST; CLOSE YOUR PAPER.

Score on Part 4 = Number right = _____

TEST VI

1. If an inscribed angle is 33, what is its intercepted arc?
2. How are tangents from an external point to a circle related?
3. What is meant by the line of centers?
4. What is meant by a common tangent?
5. What are concentric circles?
6. An angle formed by two chords intersecting within a circle is 38. How many degrees in the sum of its intercepted arc?
7. If two chords of a circle are unequal, which is the nearer to the center?
8. Bisect an arc.
9. What is the locus of a point one inch from a given point?
10. How many points not in a straight line determine a circle?
11. What is the relation between two angles inscribed in the same segment?
12. Is a chord which subtends an arc of 120 twice the length of one which subtends an arc of 60?
13. By what is a central angle measured?
14. What is the measure of an angle between two secants that intersect outside of a circle?

15. What is the relation of the opposite angles of an inscribed quadrilateral?
16. What is a segment?
17. What is a sector?
18. What is a chord?
19. Construct a tangent to a circle from a point on the circle.
20. What is an inscribed angle?

LANE-GREENE UNIT-ACHIEVEMENT TESTS IN PLANE GEOMETRY

BY RUTH O. LANE AND H. A. GREENE

Score	
Part 1	____
Part 2	____
Part 3	____
Part 4	____
Total	____

Test 4. The Circle. *Form A*

Name _____ Grade_____ Date_____

Name of School _____ City_____ State_____

General Directions. This is a test to find out how well you can work certain problems and theorems dealing with circles.

The test is in four parts. In working many of these exercises it will be helpful to make sketches of the figures needed on a piece of scratch paper.

Directions to Student for Part 1. The sample below is answered correctly. Answer the other exercises in a similar manner. Note particularly that in all the locus problems in this part of the test the locus of points is understood to be in a plane, not in three dimensions. You will be given 8 minutes for this part of the test.

SAMPLE — **Answer**

The locus of the centers of all equal circles tangent to a given circle externally is a ____*circle*____

Part 1. Locus Problems

1. All the points 5 in. from a given point in a plane are on a _____

2. The locus of points at a given distance from a given line is (a) a circle; (b) two parallel lines; (c) two points; (d) a parallel line; (e) a rectangle. (Answer *a*, *b*, *c*, *d*, or *e*.) _____

3. If two circles have the same center and chords are drawn in the larger circle which are tangent to the smaller circle, these chords, in addition to being equidistant from the center, are _____

4. The locus of points which are equidistant from two intersecting lines is (a) a circle; (b) two lines bisecting the angles; (c) a perpendicular bisector; (d) a line bisecting the angle formed by the lines; (e) four points. _____

5. Points A and B are $1\frac{1}{2}$ in. apart (A B). How many points could you mark on this paper which would be exactly 2 in. from A and also 1 in. from B? _____

6. The bisectors of all angles inscribed in a given arc of a circle (a) do not meet; (b) meet at the center of the circle; (c) meet within the circle but not at the center; (d) meet on the circumference; (e) meet without the circle. _____

7. In any triangle the locus of points halfway from the vertex to any point on the base is (a) a circle; (b) a line parallel to the base; (c) an irregular line; (d) a regular curve but not a circle; (e) a straight line not parallel to the base. _____

8. The locus of the vertices of all triangles which have a given base and a given acute vertex angle, is (a) a minor arc; (b) a semicircle; (c) a major arc; (d) a line parallel to the given base; (e) an irregular line. _____

9. The locus of the vertices of all isosceles triangles constructed on the same base is (a) any bisector of the given base; (b) a circle whose diameter is the given base; (c) two lines parallel to the given base; (d) two points on opposite sides of the given base and equidistant from its ends; (e) a perpendicular bisector of the given base. _____

10. The locus of points which are 1 in. from a given line and also 6 in. from a given point which is 5 in. from the given line is (a) no points; (b) one point; (c) two points; (d) three points; (e) four points. _____

Score on Part 1 = Number right = _____

Do not work on Part 2 until told to do so.

Directions to Student for Part 2. The sample given below is answered correctly. Answer the other exercises in a similar manner. You will be given 8 minutes for this part of the test.

	SAMPLE	Answer
	If a perpendicular is constructed to a radius at its extremity on the circle, it is a	*tangent*

Part 2. Construction Problems

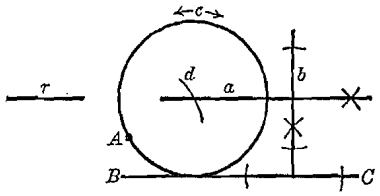
1. The number of circles which can always be constructed through three points is (a) none; (b) one; (c) two; (d) three; (e) any number. _____

2. In order to inscribe a circle in a triangle (a) two angles are bisected; (b) the mid-points of two sides are joined; (c) the perpendicular bisectors of two sides are constructed; (d) the point of intersection of the altitudes is found; (e) one side is bisected. (Answer *a*, *b*, *c*, *d*, or *e*.) _____

3. In a circle whose diameter is 8 in. the length of each side of a regular inscribed hexagon is _____

4. The number of circles which can always be drawn through four points is (a) none; (b) one; (c) two; (d) three; (e) any number. _____

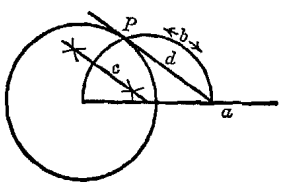
5. A circle of a given radius r has been constructed tangent to a given line BC and passing through a given point A. Write the letters *a*, *b*, *c*, *d* so that they will show the order in which the lines they represent were drawn. _____



6. The center of a circle circumscribed about a triangle lies outside the triangle if the triangle is _____

7. The correct method to use in finding the center of a circle when only an arc of it is given is (a) to construct a perpendicular bisector of one chord; (b) to construct an isosceles triangle with a chord as the base; (c) to try various points until one is found which works; (d) to construct a perpendicular to a tangent; (e) to construct perpendicular bisectors of two chords. _____

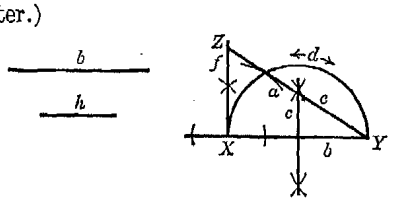
8. At a point P on a circle a tangent has been constructed to the circle. Write the letters *a*, *b*, *c*, *d* so that they will show the order in which the lines they represent were drawn. _____



9. In order to inscribe a square within a circle there must be inscribed within a semicircle a right triangle whose altitude upon the hypotenuse is a _____

10. A right $\triangle XYZ$ has been constructed using a given side b and the given altitude h upon the hypotenuse. Write the letters *a*, *b*, *c*, *d*, *e*, *f* so that they will show the order in which the lines they represent were drawn. _____

(The letter *a* represents the arc drawn when the altitude h was used as the radius of a circle of which x is the center.)



Score on Part 2 = Number right = _____

Do not work on Part 3 until told to do so.

Directions to Student for Part 3. The sample below is answered correctly. Answer the other exercises in a similar manner. You will be given 14 minutes for this part of the test.

SAMPLE	Answer
If two inscribed angles of a circle intercept the same arc of the circle, the angles are	*equal*

Part 3. Proving and Applying Theorems

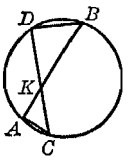
1. If a central angle is 40°, an inscribed angle intercepting the same arc is _____

2. The sides of a triangle inscribed in a circle intercept arcs of 50°, 140°, and 170°. The number of degrees in the smallest angle of the triangle is _____

3. In proving the theorem "A diameter that bisects a chord that is not a diameter is perpendicular to it" two triangles are proved congruent by showing that they have respectively equal (a) two sides and the included angle; (b) two angles and any side; (c) three sides; (d) hypotenuse and any side. _____

4. Below it has been proved that $\triangle DKB$ and $\triangle KAC$ are equiangular. *Which of the following statements is wrong, useless, or out of order? (The reasons are omitted.)* _____

a. $\angle AKC = \angle BKD$
b. $\angle A = \frac{1}{2}$ arc CB
c. $\angle D = \frac{1}{2}$ arc CB
d. $\angle A = \angle D$
e. $\angle CKB = \angle D + \angle B$
f. $\angle B = \angle C$



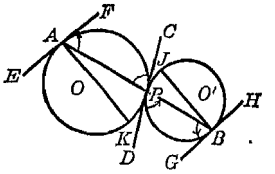
5. Two tangents to a circle from an external point are equal. One method of proving this theorem is by showing that two triangles have respectively equal (a) two sides and the included angle; (b) two angles and any side; (c) three sides; (d) the hypotenuse and an acute angle. _____

6. If the two diagonals of a quadrilateral inscribed in a circle are diameters, the figure is a _____

7. Two unequal chords are perpendicular to each other. One of the intercepted arcs is 40°. The number of degrees in the opposite arc is _____

8. Given circles O and O' tangent at P and any chord AB drawn through P, their point of tangency. Lines EF and GH are perpendicular to diameters AK and BJ. To prove $EF \parallel GH$. *Which statement in the following proof is wrong, useless, or out of order?* _____

a. $\angle CPA = \angle DPB$
b. $\angle CPA = \frac{1}{2}$ arc PA
c. $\angle DPB = \frac{1}{2}$ arc PB
d. $\frac{1}{2}$ arc $PA = \frac{1}{2}$ arc PB
e. $\angle PAF = \frac{1}{2}$ arc PA
f. $\angle PBG = \frac{1}{2}$ arc PB
g. $\angle APD = \angle BPC$
h. $\angle PAF = \angle PBG$
i. $EF \parallel GH$



9. Two intersecting chords and two secants intercept the same arcs on a circle. The angle formed by the chords is 40°, and the angle formed by the secants is 8°. The number of degrees in the larger of the intercepted arcs is _____

10. The sides of the three consecutive angles of an inscribed quadrilateral intercept arcs of 80°, 90°, and 60°, respectively. The number of degrees in the largest angle of the quadrilateral is _____

Score on Part 3 = Number right = _____

Do not work on Part 4 until told to do so.

Directions to Student for Part 4. The sample below is answered correctly. Answer the other exercises in a similar manner. You will be given 8 minutes for this part of the test.

	SAMPLE	Answer
The radii of two equal circles are		*equal*

Part 4. Miscellaneous Exercises

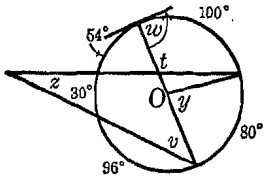
1. In the same circle or in equal circles, if the ratio of two central angles is 6 to 5, the ratio of their arcs is (a) 3 to 10; (b) 3 to 5; (c) 6 to 5; (d) 2 to 5; (e) 12 to 5. _____

2. If two chords of a circle are the same distance from the center, they are _____

3. An inscribed angle is formed by two _____

4. If two chords are constructed perpendicular to a radius of a circle, the chord thus constructed at the greater distance from the center is the _____

5. O is the center of this circle. The number of degrees in $\angle y$ is _____



6. In the figure for exercise 5 the number of degrees in $\angle v$ is _____

7. In the figure for exercise 5 the number of degrees in $\angle w$ is _____

8. In the figure for exercise 5 the number of degrees in $\angle z$ is _____

9. In the figure for exercise 5 the number of degrees in $\angle t$ is _____

10. If in two different circles the length of one degree of their arcs is not the same, then the circles are _____

11. If the distance between the centers of two circles equals the sum of their radii, the number of common tangents which can be drawn is _____

12. A trapezoid inscribed in a circle always (a) has unequal diagonals; (b) is isosceles; (c) has diagonals which pass through the center; (d) has three sides equal; (e) contains a right angle. _____

13. If a wheel revolves five times per minute, the number of degrees it revolves in one second is _____

14. The opposite angles of a quadrilateral inscribed in a circle are always (a) unequal; (b) equal; (c) supplementary; (d) complementary; (e) right angles. _____

15. The lines of a triangle which meet at a point two thirds of the distance from the vertex to the opposite side are the _____

END OF THE TEST; CLOSE YOUR PAPER.

Score on Part 4 = Number right = _____

LANE-GREENE UNIT-ACHIEVEMENT TESTS IN PLANE GEOMETRY

BY RUTH O. LANE AND H. A. GREENE

Score	
Part 1	____
Part 2	____
Part 3	____
Part 4	____
Total	____

Test 5. Proportion and Similar Polygons. *Form A*

Name _____ Grade_____ Date_____

Name of School _____ City_____ State_____

General Directions. This is a test to find out how well you can work certain problems and theorems dealing with proportion and similar polygons. The test is in four parts. In working many of these exercises it will be helpful to make sketches of the figures needed on a piece of scratch paper.

Directions to Student for Part 1. The sample below is answered correctly. Answer other exercises in a similar manner. You will be given 4 minutes for this part of the test.

SAMPLE	Answer
If $\frac{a}{b}=\frac{c}{d}$ and $a=c$, it follows that b equals	d

Part 1. Proportion

1. The product of the means of a proportion is equal to the product of the _____

2. If $\frac{2}{5}=\frac{12}{x}$, the value of x is _____

3. The ratio of $5\frac{1}{2}$ to 22 can be expressed as the ratio of 1 to _____

4. If $\frac{h}{k}=\frac{g}{m}$, the equation which expresses the same thing in the form of products is _____

5. The mean proportional between 9 and 16 is _____

6. We may say that a proportion is a statement of two equal ratios or of two equal _____

7. If $\frac{x}{y}=\frac{2}{3}$, it follows that $\frac{x^3}{y^3}$ equals _____

8. If a is to x as x is to b, it follows that x equals _____

9. If the number of degrees in the angles of a triangle are to each other as 3, 4, and 5, the number of degrees in the smallest angle of the triangle is _____

10. The proportion expressing that $x=\frac{ab}{c}$ and having x for the last term is _____

Score on Part 1 = Number right = _____

Do not work on Part 2 until told to do so.

Directions to Student for Part 2. The sample below is answered correctly. Answer other exercises in a similar manner. You will be given 8 minutes for this part of the test.

SAMPLE	Answer
If the lengths of four lines are 2, 4, 3, and 6, one of the proportions which can be made to express the relationship between the lengths is	$\frac{2}{4}=\frac{3}{6}$

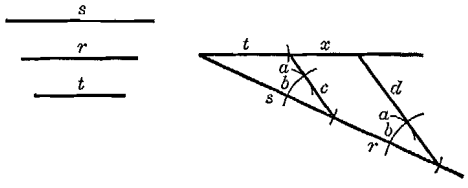
Part 2. Proportional Lines

1. If a line parallel to the base of a triangle divides one side into segments which are 6 and 9 and the other side into segments which are respectively 10 and x, the length of x is _____

2. A room is 24 ft. long and 20 ft. wide. If the scale used in making a drawing of the room is $\frac{1}{4}$ in. to 1 ft., the dimensions of the drawing are _____

3. The sides of a triangle are in the ratio 5 : 6 : 7 and the perimeter is 72. The length of the longest side is _____

4. In this figure x has been constructed in the same ratio to t as r is to s. Write the letters a, b, c, d so that they will show the order in which the lines they represent were drawn. _____

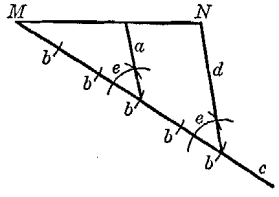


5. The sides of a triangle are 15, 10, and 20. The lengths of the segments into which the side 20 is divided by the bisector of the angle opposite are _____

6. If four parallel lines are cut by two transversals and the segments of one transversal are 1, 3, and 4, and the segment on the second transversal corresponding to the segment which is 1 on the first transversal is 5, the other two segments on the second transversal are _____

7. The line MN has been divided into the ratio 3 to 2. Write the letters a, b, c, d, e to show the order in which the lines they represent were drawn. _____

(If the same letter is used more than once on the drawing, it indicates a similar construction, and the letter need be given but once in the answer.)



8. If the line MN in the figure of exercise 7 is 25 units in length, the segments into which it is divided by the line a are _____

9. The base of any triangle and the line dividing the other two sides in the ratio of 1 to 3 are _____

10. The bisector of the exterior angle adjacent to the largest angle of a triangle whose sides are 12, 10, and 5 divides the opposite side externally into segments which are _____

Score on Part 2 = Number right = _____

Do not work on Part 3 until told to do so.

Directions to Student for Part 3. In working part of these exercises it will be helpful to make sketches of the figures needed on a piece of scratch paper. The sample below is answered correctly. Answer the other exercises in a similar manner. You will be given 16 minutes for this part of the test.

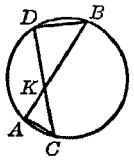
SAMPLE

Any two isosceles right triangles are _____ Answer *similar*

Part 3. Proving and Applying Theorems

1. Two polygons are always similar if both are (a) rectangles; (b) rhombuses; (c) parallelograms; (d) squares; (e) isosceles trapezoids. (Answer *a*, *b*, *c*, *d*, or *e*.) _____

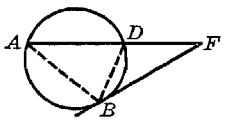
2. If AK is 4, KB is 15, and DK is 6, then KC is _____



3. The statement below which is *not* sufficient to prove two triangles similar is (answer *a*, *b*, *c*, *d*, or *e*) _____

a. The corresponding sides are proportional.
b. An angle of one equals an angle of the other and the including sides are proportional.
c. They are right triangles with a side of one equal to twice the length of a side of the other.
d. Their sides are respectively perpendicular.
e. They are similar to the same triangle.

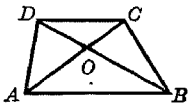
4. If a secant and a tangent are drawn to a circle from an external point, the tangent is a mean proportional between the whole secant and its external segment. *The two triangles which must be proved similar in the proof of this theorem are* _____



5. The altitude upon the hypotenuse of a right triangle is 9, and one of the segments of the hypotenuse is 3. The length of the hypotenuse is _____

6. If the diagonals of a quadrilateral divide each other proportionally, the quadrilateral is a trapezoid. *Which of the following statements in the proof of this theorem is wrong, useless, or out of order?* (*The reasons are omitted.*) _____

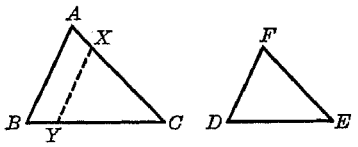
a. $DO/OB = CO/OA$
b. $\angle DOC = \angle AOB$
c. $\triangle DOC \sim \triangle AOB$



d. $\angle CDO = \angle OAB$
e. $DC \parallel AB$
f. $ABCD$ is a trapezoid

7. Two triangles are similar if they are mutually equiangular. *Which of the following statements in the proof of this theorem is wrong, useless, or out of order?* (*The explanation of superposition and reasons are omitted.*) _____

a. $\angle D = \angle XYC = \angle B$
b. $XY \parallel AB$
c. $CA/CX = CB/CY$
d. $CA/EF = CB/ED$



e. $\angle F = \angle CXY = \angle A$
f. $CA/EF = AB/FD$ (similar proof)
g. $CA/EF = AB/FD = CB/ED$
h. $\triangle ABC \sim \triangle DEF$

8. Angle C of triangle ABC is a right angle. From G any point on BC, GE is drawn perpendicular to AB. The three equal ratios on this figure are _____

9. If in a railroad curve (arc of a circle) the distance from the mid-point of the arc of a 200-foot chord to the chord is 10 ft., the radius of the circle is _____

10. RA and MB, the altitudes drawn from R and M of $\triangle RMN$, meet at O. OA is to BN as AM is to _____

Score on Part 3 = Number right = _____

Do not work on Part 4 until told to do so.

TEST IX

FACTS

1. Two lines parallel to the same line are how related to each other?

2. If two parallel lines are cut by a transversal, what pairs of angles are equal?

3. State five relationships concerning a parallelogram.

4. To what is an exterior angle of a triangle equal?

5. State five ways of showing that two angles are equal.

6. What is the most common way of proving two angles equal?

7. How are two lines related that are perpendicular to the same line?

8. Give three ways of proving any two triangles congruent.

9. From what is a point on a mid-perpendicular equidistant?

10. On what does a point lie that is equidistant from the sides of an angle?

11. Three or more parallels that intercept equal segments on one transversal intercept what on any other transversal?

12. What may be said of a line that bisects on side of a triangle and is parallel to another side?

13. If a line bisects two sides of a triangle, it is how related to the third side?

14. Where do the medians of a triangle meet?

15. How many degrees in the sum of the interior angles of a polygon?

16. How many degrees in the exterior angles of a polygon?

17. To what is the side opposite a 30° angle in a right triangle equal?

18. How many points not in a straight line determine a circle?

19. State three facts concerning two equal chords.

20. State three facts concerning unequal chords.

21. How are tangents from a fixed external point to a circle related?

22. What is the measure of an inscribed angle?

23. What is the measure of a central angle?

24. An angle formed by two intersecting chords is measured by what?

25. An angle between two secants is measured by what?

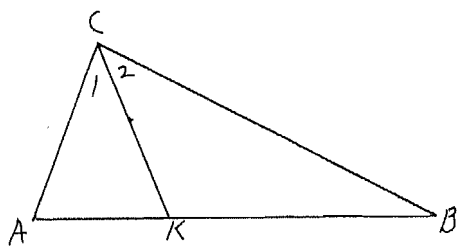
26. What is the relation between opposite angles of an inscribed quadrilateral?

27. Angles inscribed in the same segment are how related to each other?

28. If a line is parallel to one side of a triangle, it does what to the other two sides?

29. If a line divides two sides of a triangle proportionally it is how related to the third side?

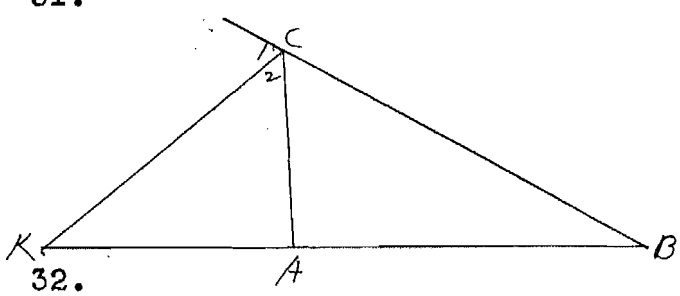
30.



Given $\angle 1 = \angle 2$.

State a proportion.

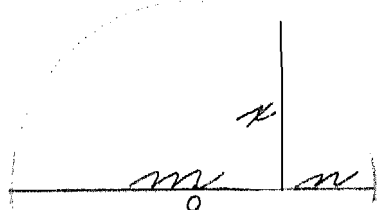
31.



Given $\angle 1 = \angle 2$.

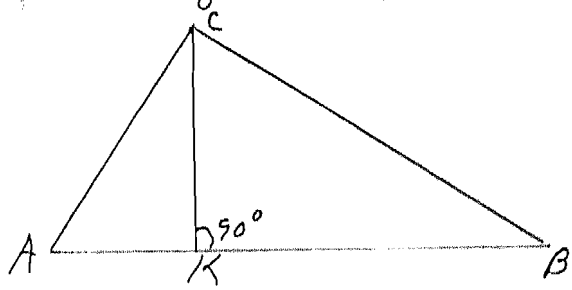
State a proportion.

32.



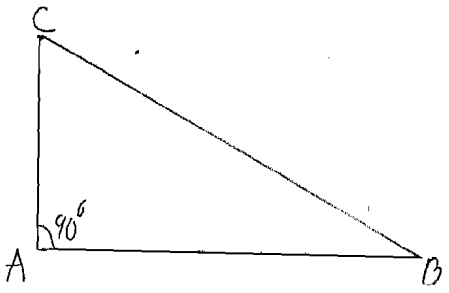
How is X related to M and N?

33.
34.
35.

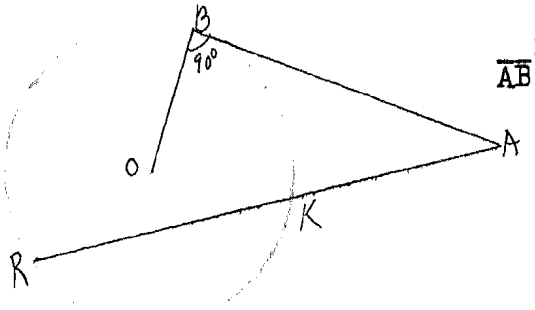


$\angle ACB$ is a right angle.

State three proportions.

36. 

$\overline{BC}^2 = ?$

37. 

$\overline{AB}^2 = ?$

38. Fill in blanks:

The _____ of the segments of one intersecting chord _____ the _____ of the segments of the other.

39. Complete:

The perimeters of two similar polygons are to each other as any _____ _____.

40. What do two parallel lines cut off on a circle?

TEST X

DEFINITIONS

Define:

1. An acute angle.
2. An obtuse angle.
3. Perpendicular lines.
4. Degree of angle.
5. Complementary angles.
6. Supplementary angles.
7. Adjacent angles.
8. Acute triangle.
9. Obtuse triangle.
10. Isoceles triangle.
11. Equilateral triangle.
12. Right triangle.
13. Congruent figures.
14. Auxillary lines.
15. Altitude of triangle.
16. Median of triangle.
17. Parallel lines.
18. Transversal.
19. Exterior angle of a triangle.
20. Parallelogram.
21. Trapezoid.
22. Median of a trapezoid.
23. Midperpendicular.
24. Isosceles trapezoid.
25. Regular Polygon.
26. Central angle.

27. Inscribed angle.

28. Arc.

29. Secant.

30. Concentric circles.

31. Tangent.

32. Degree of arc.

33. Segment of a circle.

34. Sector of a circle.

35. Line of centers.

36. Extremes (proportion)

37. Means (proportion)

38. Alternation (proportion)

39. Inversion (proportion)

40. Addition (proportion)

41. Subtraction (proportion)

42. Similar Polygons

43. Common tangent

44. Minor arc

45. Circumference

46. Diagonal of a polygon

47. Hexagon.

48. Angle bisector

49. Alternate interior angles

50. Protractor

TEST XI

CONSTRUCTIONS

1. Construct a triangle whose sides are one inch, two inches, and one and one-half inches.

2. Construct a perpendicular to a line from a point on the line.

3. Construct a perpendicular to a line from a point off the line.

4. Construct an angle equal to a given angle.

5. Through a point construct a line parallel to a given line.

6. Bisect an angle.

7. Bisect a given line segment.

8. Divide a given line into three equal parts.

9. Construct a circle through three given points.

10. Bisect an arc.

11. Construct a tangent to a circle from an external point.

12. Construct locus of a point one inch from a given point.

13. Construct locus of a point equidistant from two points.

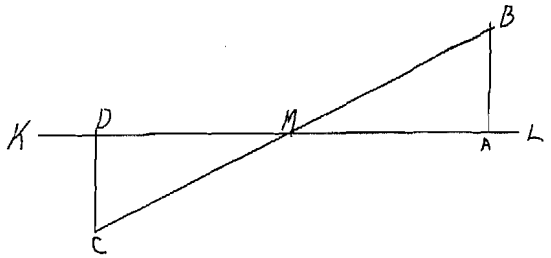
14. Construct locus of a point equidistant from two intersecting lines.

15. Construct a 60° angle.

16. Draw to scale a triangle whose sides are 10', 20', and 16', using scale of 1" = 4'.

TEST XII

1

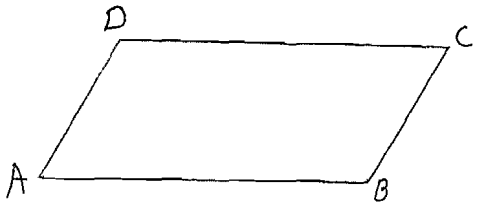


Given: $BA \perp LK$
M midpoint of DA
$CD \perp LK$

Prove: $\angle B = \angle C$

Proof:

2

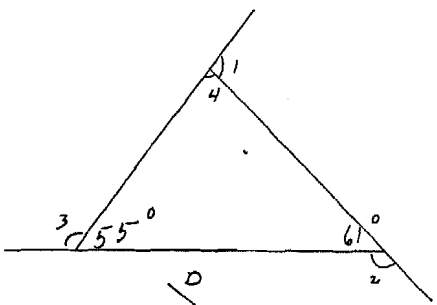


Given: Parallelogram ABCD

Prove: $\angle A = \angle C$

Proof:

4 3



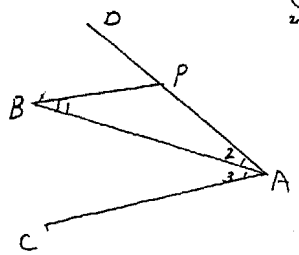
3. Given: $AK \perp DB$

$CM \perp DB$

Prove: $AK = CM$

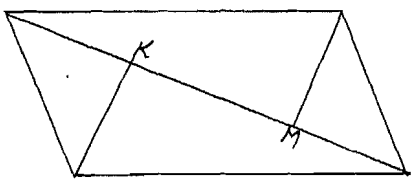
Proof:

5 4



4. Given: Triangle with three exterior angles.
Prove: Find number of degrees in angles 1, 2 3, and 4.
Proof:

3 5



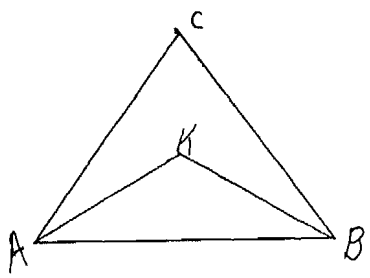
6. Given: BA bisects DAC

BP ∥ CA

Prove:

Proof:

6



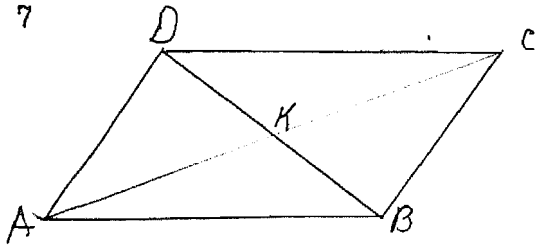
Given: $AC = CB$

AK bisects $\angle A$

BK bisects $\angle B$

Prove: $AK = KB$

7

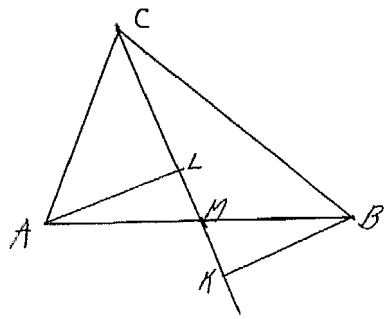


Given: Parrllogram ABCD

Prove: The diagonals intersect each other at their midpoint.

Proof:

8



Given: AL⊥ CK

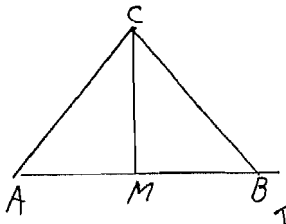
BK ⊥CK

M midpoint of AB

Prove: AL BK

Proof:

9

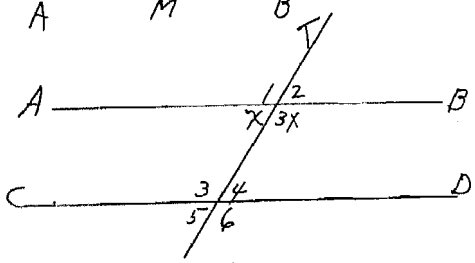


Given: CM a median ⊥AB

Prove: Triangle ABC isosceles.

Proof:

10



Given: AB//CD

Prove: Find size of angles x,3x,1,2,3,4,5,and 6

Proof:

Value assigned to each problem:

1--12	points	6---6	points
2--15	points	7--18	points
3--12	points	8--12	points
4---8	points	9--15	points
5--15	points	10---8	points

Total possible points:121

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